New Topic:
Variation
- Directly $\Rightarrow$ Multiplication
- Inversely $\Rightarrow$ Division

$y$ varies directly as $x$. $\Rightarrow y = kx$
$y$ varies inversely as $x$. $\Rightarrow y = \frac{k}{x}$
y varies directly as $x^2$.  

$$y = k \cdot x^2$$

$y$ is 100 when $x = 2$.

$$100 = k \cdot 2^2$$

$$k = 25$$

Find $y$ when $x$ is 4.

$$y = 25 \cdot 4^2$$

$$y = 25 \cdot 16$$

$$y = 400$$

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y varies inversely as the cube of $x$.

$$y = \frac{k}{x^3}$$

$y$ is 10 when $x$ is 2.

$$10 = \frac{k}{2^3}$$

$$k = 80$$

Find $y$ when $x$ is 4.

$$y = \frac{80}{4^3}$$

$$y = \frac{80}{64}$$

$$y = 1.25$$
y varies directly as \( \sqrt{x} \).

y is 80 when \( x \) is 64.

Find \( y \) when \( x \) is 100.

\[
y = 100
\]

\[
y = 10 \sqrt{100}
\]

\[
y = 10 \cdot 10 = 100
\]

y is 25 when \( x \) is 2.

y varies inversely as \( x^2 \).

Find \( y \) when \( x = 10 \).

\[
y = \frac{100}{x^2}
\]

\[
y = \frac{100}{10^2} = \frac{100}{100} = 1
\]

When \( x \) is 10, \( y \) is 1.
Joint Variation

$Z$ varies directly as $x$, and inversely as $y^2$.

$Z = \frac{k \cdot x}{y^2}$

$Z = 20$ when $x$ is 8 and $y$ is 2.

Find $Z$ when $x$ is 16 and $y$ is 4.

$Z = \frac{10 \cdot x}{y^2}$

$Z = \frac{10 \cdot 16}{4^2}$ $Z = 10$  \[ k = 10 \]

$Z$ varies directly as the square root of the sum of $x^2$ and $y^2$.

$Z = k \cdot \sqrt{x^2 + y^2}$

$50 = k \cdot \sqrt{3^2 + 4^2}$

$50 = k \cdot 5$ $\Rightarrow k = 10$

Find $Z$ when $x$ is 6 and $y$ is 8.

$Z = k \cdot \sqrt{6^2 + 8^2}$ $Z = 10 \cdot \sqrt{100}$

$Z = 10 \cdot 10$ $Z = 100$
Z varies inversely as square root of the difference of $x^2$ and $y^2$.

Z is 40 when $x=10$ and $y=6$

Find Z when $x=5$ and $y=3$.

\[ Z = \frac{k}{\sqrt{x^2 - y^2}} \]

\[ 40 = \frac{k}{\sqrt{10^2 - 6^2}} \]

\[ 40 = \frac{320}{\sqrt{64}} \]

\[ k = 320 \]

\[ Z = \frac{320}{\sqrt{5^2 - 3^2}} = \frac{320}{\sqrt{16}} = \frac{320}{4} \]

\[ Z = 80 \]

The weight of rectangular beam varies jointly as its width and square of its length. Beam is 100 when width is 5 and length is 2.

Find the beam when width is 3 and length is 6.

\[ B = k \cdot W \cdot L^2 \]

\[ 100 = k \cdot 5 \cdot 2^2 \]

\[ 100 = 20k \]

\[ k = 5 \]

\[ B = 5 \cdot 3 \cdot 6^2 \]

\[ B = 540 \]
The intensity of a light varies inversely as the square of its distance from the light source.

\[ I = \frac{K}{D^2} \]

when you are 2 ft from the source of the light, the intensity is 80 units.

Find the intensity when you are \( \frac{1}{2} \) ft from the source.

\[ 80 = \frac{K}{2^2} \quad K = 320 \]

\[ I = \frac{320}{D^2} \quad I = \frac{320}{(\frac{1}{2})^2} = \frac{320}{\frac{1}{4}} \]

\[ I = 1280 \text{ units} \]

Volume of a cone varies directly as the product of its height and square of its radius.

\[ V = K \cdot h \cdot r^2 \]

Volume of cone is \( 32\pi \) in\(^3\)

when radius is 4 in. and height is 6 in.

Find the volume of the cone when radius is 8 in. and height is 9 in.

\[ V = \frac{\pi}{3} h r^2 \quad K = \frac{\pi}{3} \quad 32\pi = K \cdot 6 \cdot 4^2 \]

\[ V = \frac{\pi}{3} \cdot 9 \cdot 8^2 \]

\[ V = 192\pi \text{ in}^3 \]
Voltage varies inversely as Resistance. If the voltage is 40 amperes, when the resistance is 270 ohms. Find the voltage when the resistance is 150 ohms.

\[
V = \frac{K}{R}
\]

\[
V = \frac{10800}{R}
\]

\[
40 = \frac{K}{270}
\]

\[
V = \frac{10800}{150}
\]

\[
V = 72 \text{ Amps}
\]

\[
k = 10800
\]

The weight of an object on or above the surface of Earth varies inversely as the square of the distance of the object to the center of Earth.

160 pound person moves 200 miles above Earth, find his/her weight.

\[
W = \frac{K}{D^2}
\]

\[
160 = \frac{K}{4000^2}
\]

\[
W = \frac{25600000000}{D^2}
\]

\[
W = \frac{25600000000}{4200^2}
\]

\[
W \approx 145 \text{ pounds}
\]
Zero-Product Rule:
If \( A \cdot B = 0 \), then \( A = 0 \) or \( B = 0 \) or maybe both.

Solve \((x-1)(x+8) = 0\)
By Z.P.R. \( x-1 = 0 \) or \( x+8 = 0 \)
\( x = 1 \) or \( x = -8 \)
\( \{ -8, 1 \} \)

Solve \((2x+3)(3x-5)(3x+5) = 0\)
RHS = 0, LHS is in Factored Form
Use Z.P.R.
\( 2x + 3 = 0 \) or \( 3x - 5 = 0 \) or \( 3x + 5 = 0 \)
\( x = \frac{-3}{2} \) \( x = \frac{5}{3} \) \( x = \frac{-5}{3} \)
Solution set \( \{ \frac{-3}{2}, \pm \frac{5}{3} \} \)
Solve $x^2 - 15 = 2x$ by Factoring

$x^2 - 15 - 2x = 0$

$x^2 - 2x - 15 = 0$

$(x - 5)(x + 3) = 0$

by Z.P.R. $x-5=0$ or $x+3=0$

$x=5$  $x=-3$

$\{ -3, 5 \}$

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Solve by Factoring

$2x^2 - 5 = -3x$

$2x^2 - 5 + 3x = 0$

$2x^2 + 3x - 5 = 0$

$(2x + 5)(x - 1) = 0$

by Z.P.R. $2x+5=0$ or $x-1=0$

$x=\frac{-5}{2}$  $x=1$

$\{ -\frac{5}{2}, 1 \}$
Solve \( x^2 + (x+2)^2 = 100 \)

\[
\begin{align*}
\chi^2 + (x+2)(x+2) &= 100 \\
\text{FOIL, Simplify} \\
\chi^2 + x^2 + 2x + 2x + 4 - 100 &= 0 \\
2\chi^2 + 4x - 96 &= 0 \\
\text{Divide by 2 to reduce} \\
\chi^2 + 2\chi - 48 &= 0 \\
(\chi + 8)(\chi - 6) &= 0 \\
\text{By Z.P.R.} \quad \chi + 8 = 0 \quad \text{or} \quad \chi - 6 = 0 \\
\chi &= -8 \quad \chi = 6 \\
\{ -8, 6 \}
\end{align*}
\]

Solving \( ax^2 + bx + c = 0, \ a \neq 0 \) by

**Quadratic Formula** \( \chi = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \)

\[
2\chi^2 + 3\chi - 5 = 0 \\
\chi = \frac{-3 \pm \sqrt{(3)^2-4(2)(-5)}}{2(2)} = \frac{-3 \pm \sqrt{9+40}}{4} \\
\chi = \frac{-3 \pm 7}{4} = \frac{-3 \pm \sqrt{49}}{4} = \frac{-3 \pm 7}{4} = \frac{-3 + 7}{4} = \frac{4}{4} = \boxed{1} \quad \chi = \frac{-3 - 7}{4} = \frac{-10}{4} = \boxed{-\frac{5}{2}} = -\frac{3 \pm 7}{4}
\]
Solve by Quadratic Formula

\[ 3x^2 - 2x - 1 = 0 \]

\[ a=3 \quad b=-2 \quad c=-1 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{2 \pm \sqrt{4 + 12}}{6} \]

\[ \{ x = \frac{-1}{3}, 1 \} = \frac{2 \pm \sqrt{16}}{6} = \frac{2 \pm 4}{6} \]