Ch. 11:
Square Root Method:
If \( x^2 = K \), then \( x = \pm \sqrt{K} \)

Solve
1) \( (2x - 5)^2 = 80 \)
   \[
   2x - 5 = \pm \sqrt{80} \\
   2x = 5 \pm \sqrt{16\sqrt{5}} \\
   x = \frac{5 \pm 4\sqrt{5}}{2} \\
   \{ \frac{5 \pm 4\sqrt{5}}{2} \}
   \]
2) \( (3x + 4)^2 = -50 \)
   \[
   3x + 4 = \pm \sqrt{-50} \\
   3x = -4 \pm 5\sqrt{2} \sqrt{-1} \\
   x = \frac{-4 \pm 5\sqrt{2} i}{3} \\
   \{ \frac{-4 \pm 5\sqrt{2} i}{3} \}
   \]
Make a Perfect Square:

1) \( x^2 + 14x + 49 = (x + 7)^2 \)

2) \( x^2 - 5x + \frac{25}{4} = \left(x - \frac{5}{2}\right)^2 \)

3) \( x^2 - \frac{4}{3}x + \frac{4}{9} = \left(\frac{1}{2} - \frac{2}{3}\right)^2 \)

4) \( x^2 + \frac{7}{2}x + \frac{49}{16} = \left(\frac{1}{2} + \frac{7}{4}\right)^2 \)

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USE completing the Square method to Solve:

1) \( x^2 + 6x - 11 = 0 \)

\( x^2 + 6x + 9 = 11 + 9 \)

\( \frac{1}{2} \cdot 6 = 3, \ 3^2 = 9 \)

\( (x + 3)^2 = 20 \)

By S.R.M.

\( x + 3 = \pm \sqrt{20} \)

\( x + 3 = \pm \sqrt{4 \cdot 5} \)

\( x = -3 \pm 2\sqrt{5} \) \( \{ -3 \pm 2\sqrt{5} \} \)

2) \( x^2 + 7x + 10 = 0 \)

\( x^2 + 7x + \frac{49}{4} = -10 + \frac{49}{4} \)

\( \frac{1}{2} \cdot 7 = \frac{7}{2}, \ (\frac{7}{2})^2 = \frac{49}{4} \)

\( (x + \frac{7}{2})^2 = \frac{9}{4} \)

\( x + \frac{7}{2} = \pm \frac{3}{2} \)

\( x = -\frac{7}{2} \pm \frac{3}{2} \)

\( x = -2, -5 \) \( \{ -2, -5 \} \)
3) \(3x^2 - 2x - 5 = 0\)

Divide by 3, then move the constant term to the other side.

\[
\frac{1}{3} \cdot 2 = \frac{1}{3}, \quad \left(\frac{1}{3}\right)^2 = \frac{1}{9}
\]

\[
\left(\frac{x - \frac{1}{3}}{3}\right)^2 = \frac{16}{9}
\]

by S.R.M.

\[
x - \frac{1}{3} = \pm \frac{4}{3}
\]

\[
x = \frac{1}{3} \pm \frac{4}{3}
\]

\[
x = \frac{1+4}{3}, \quad x = \frac{1-4}{3}
\]

\[
x = \frac{5}{3}, \quad x = -1
\]

\[
\left\{ \frac{5}{3}, -1 \right\}
\]

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Solve by Completing the Sqr method:

\(x^2 - 10x + 41 = 0\)

\(x^2 - 10x + 25 = -41 + 25\)

\[
\left(x - 5\right)^2 = -16
\]

By S.R.M.

\[
x - 5 = \pm \sqrt{-16}
\]

\[
x = 5 \pm 4i
\]

\[
\left\{ 5 \pm 4i \right\}
\]
use Q-formula to solve

\[ 4x^2 - x - 5 = 0 \]

\[ a=4 \quad b=-1 \quad c=-5 \]

\[ b^2-4ac = (-1)^2 - 4(4)(-5) = 1 + 80 = 81 \]

\[ x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{81}}{8} = \frac{1 \pm 9}{8} \]

\[ x = \frac{1 + 9}{8} = \frac{10}{8} = \frac{5}{4} \]

\[ x = \frac{1 - 9}{8} = \frac{-8}{8} = -1 \]

\[ \{-1, \frac{5}{4}\} \]

Area of a rectangle is 30 ft^2.
The length is 1 ft longer than 3 times its width.

1) Draw & label

2) Find the eqn for A=30

\[ LW = 30 \]

\[ (3x+1)x = 30 \]

3) Use Q-formula to solve

\[ b^2-4ac = 1^2 - 4(3)(-30) \]

\[ x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{361}}{6} \]

\[ x = \frac{-1 \pm 19}{6} \]

\[ x = \frac{18}{6} \quad x = \frac{-20}{6} \]

4) Give its dimensions.

3 ft by 10 ft.
Area of a rectangle is 28 cm². The length is 1 cm shorter than twice its width. Find its dimensions.

\[ x(2x-1)=28 \]
\[ 2x^2 - x - 28 = 0 \]
Completing the square method
\[ x^2 - \frac{1}{2}x + \frac{1}{16} = 14 + \frac{1}{16} \]
\[ \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}, \quad \left(\frac{1}{4}\right)^2 = \frac{1}{16} \]
\[ \left(x - \frac{1}{4}\right)^2 = \frac{225}{16} \]
\[ x = \frac{16}{4}, \quad x = 4 \]
\[ x = -\frac{14}{4}, \quad 4 \text{cm by 7cm} \]

The sum of squares of two cons. odd integers is 34. Find all such odd integers.

\[ x \cdot (x+2) \]
\[ x^2 + (x+2)^2 = 34 \]
\[ x^2 + x^2 + 4x + 4 = 34 \]
\[ 2x^2 + 4x - 30 = 0 \]
\[ x^2 + 2x - 15 = 0 \]
\[ x^2 + 2x + 1 = 16 \]
\[ x = -1 \pm 4 \]
\[ x = 3 \quad \text{or} \quad x = -5 \]
The sum of the reciprocal of two consecutive integers is $\frac{5}{6}$. Find such integers. Integers: $x, x+1$

$$\frac{1}{x} + \frac{1}{x+1} = \frac{5}{6}$$

Reciprocal: $\frac{1}{x}, \frac{1}{x+1}$

$$\text{LCM} = 6x(x+1)$$

$$6(x+1) + 6x = 5x(x+1)$$

$$6x + 6 + 6x = 5x^2 + 5x$$

$$5x^2 - 7x - 6 = 0$$

$$a = 5, b = -7, c = -6$$

$$b^2 - 4ac = (-7)^2 - 4(5)(-6)$$

$$= 169$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{169}}{10}$$

$$x = \frac{7 \pm 13}{10}$$

$$x = \frac{7 + 13}{10} = 2$$

$$x = \frac{7 - 13}{10} = -\frac{3}{5}$$

Exam 3: Tuesday
More details tomorrow
SG 16 due tomorrow.
SG 17 due Monday

Bring a Scientific Calculator with you starting tomorrow.

Use discriminant to determine the type of solutions for

$$a x^2 + b x + c = 0$$

$$4x^2 + 12 = 3x$$

$$a = 4, b = -3, c = 12$$

$$b^2 - 4ac = (-3)^2 - 4(4)(12) = -117$$

$$= -117$$

Two complex solns.
Find a quadratic eqn in the form of \( ax^2 + bx + c = 0 \) with Solutions 2 \( \pm \sqrt{3} \).

\[
\begin{align*}
\chi &= 2 + \sqrt{3} \\
\chi &= 2 - \sqrt{3} \\
\chi - 2 - \sqrt{3} &= 0 \\
\chi - 2 + \sqrt{3} &= 0
\end{align*}
\]

\[
\begin{align*}
(\chi - 2 - \sqrt{3})(\chi - 2 + \sqrt{3}) &= 0 \\
(A - B)(A + B) &= 0 \\
A^2 - B^2 &= 0 \\
(\chi - 2)^2 - (\sqrt{3})^2 &= 0 \\
\chi^2 - 4\chi + 1 &= 0
\end{align*}
\]

Find a quadratic eqn in the form of \( ax^2 + bx + c = 0 \) with Solutions \(-4 \pm 3\sqrt{2}\).

\[
\begin{align*}
\chi &= -4 + 3\sqrt{2} \\
\chi &= -4 - 3\sqrt{2} \\
\chi + 4 - 3\sqrt{2} &= 0 \\
\chi + 4 + 3\sqrt{2} &= 0
\end{align*}
\]

\[
\begin{align*}
\left(\frac{\chi + 4 - 3\sqrt{2}}{A}\right)\left(\frac{\chi + 4 + 3\sqrt{2}}{B}\right) &= 0 \\
\left(\frac{\chi + 4}{A}\right)^2 - (3\sqrt{2})^2 &= 0 \\
\chi^2 + 8\chi - 2 &= 0 \\
\chi^2 + 8\chi + 16 - 9\cdot 2 &= 0
\end{align*}
\]
Find a quadratic eqn in the form of $ax^2 + bx + c = 0$ with Solutions $3 \pm 4i$

$x = 3 + 4i$  \hspace{1cm} x = 3 - 4i$

$x - 3 - 4i = 0$  \hspace{1cm} x - 3 + 4i = 0

\[
\frac{x - 3}{(x - 3 - 4i)(x - 3 + 4i)} = 0
\]

\[
(x - 3)^2 - (4i)^2 = 0
\]

\[
x^2 - 6x + 9 - (16i^2) = 0
\]

\[
x^2 - 6x + 25 = 0
\]

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Find a quadratic eqn in the form of $ax^2 + bx + c = 0$ with Solutions $-\frac{1}{2} \pm \frac{5}{2}i$

$x = -\frac{1}{2} + \frac{5}{2}i$  \hspace{1cm} x = $-\frac{1}{2} - \frac{5}{2}i$

$2x = -1 + 5i$  \hspace{1cm} 2x = -1 - 5i

$2x + 1 - 5i = 0$  \hspace{1cm} 2x + 1 + 5i = 0

\[
(2x + 1 - 5i)(2x + 1 + 5i) = 0
\]

\[
x^2 + 4x + 26 = 0
\]

\[
(2x + 1)^2 - (5i)^2 = 0
\]

\[
x^2 - 4x + 1 + 25 = 0
\]
Solving equations that are in Quadratic Form:

\[ x^4 - 5x^2 - 36 = 0 \]

Let \( u = x^2 \), \( u^2 = x^4 \)

\[ u^2 - 5u - 36 = 0 \]

\[ (u - 9)(u + 4) = 0 \]

\[ u = 9 \quad u = -4 \]

\[ x^2 = 9 \quad x^2 = -4 \]

\[ x = \pm 3 \quad x = \pm 2i \]

Solve by making a Subs.

\[ x^6 - x^3 - 2 = 0 \]

Let \( u = x^3 \), \( u^2 = x^6 \)

\[ u^2 - u - 2 = 0 \]

\[ (u - 2)(u + 1) = 0 \]

\[ u = 2 \quad u = -1 \]

\[ x^3 = 2 \quad x^3 = -1 \]

\[ x = \sqrt[3]{2} \quad x = \sqrt[3]{-1} \]

\[ x = -1 \]
\[ \sqrt{x} + \sqrt{x} = 6 = 0 \]

\[ \frac{1}{4} + \frac{1}{2} = 6 = 0 \]

Let \( u = \sqrt{x} = \frac{1}{4} \), \( u^2 = \sqrt{x} = \frac{1}{2} \)

\[ u + u^2 = 6 = 0 \]

\[ u^2 + u - 6 = 0 \]

\[ (u + 3)(u - 2) = 0 \]

\[ u = -3 \quad u = 2 \]

\[ \sqrt{x} = -3 \quad \sqrt{x} = 2 \]

\[ x = 16 \]

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Solve by making subs.

\[ (x^2 + 6x)^2 - 7(x^2 + 6x) + 12 = 0 \]

Let \( u = x^2 + 6x \quad u = 4 \quad u = 3 \)

\[ u^2 - 7u + 12 = 0 \]

\[ (u - 4)(u - 3) = 0 \]

\[ x^2 + 6x = 4 \quad x^2 + 6x = 3 \]

\[ x = \quad x = \]

\[ u = 4 \quad u = 3 \]

\[ x = \quad x = \]
\[
\left\{ \begin{array}{c}
\chi^{2/3} - 3\chi^{1/3} - 10 = 0 \\
\text{Let } u = \chi^{1/3}, \quad u^2 = \chi^{2/3}
\end{array} \right. \\
\Rightarrow u^2 - 3u - 10 = 0 \\
\quad (u - 5)(u + 2) = 0 \\
\quad u = 5, \quad u = -2
\]
\[
\begin{array}{c}
u = 5 \\
\chi^{1/3} = 5 \\
3\sqrt[3]{\chi} = 5 \\
\chi = 125
\end{array}
\quad \begin{array}{c}
u = -2 \\
\chi^{1/3} = -2 \\
3\sqrt[3]{\chi} = -2 \\
\chi = -8
\end{array}
\quad \left\{-8, 125\right\}
\]

\[
\left\{ \begin{array}{c}
\chi^{2/5} - 8\chi^{1/5} + 16 = 0 \\
\text{Let } u = \chi^{1/5}, \quad u^2 = \chi^{2/5}
\end{array} \right. \\
\Rightarrow u^2 - 8u + 16 = 0 \Rightarrow u = 4
\]
\[
\begin{array}{c}
u = 4 \\
\chi^{1/5} = 4 \\
5\sqrt[5]{\chi} = 4
\end{array}
\quad \begin{array}{c}
\chi = 4^5 \\
\chi = 1024
\end{array}
\]
\[
\left\{10, 24\right\}
\]