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|---------------------------|--|
| Trigonometry | Name: [REDACTED] |
| Study Guide 15 | Class: _____ |
| Due Date: <u>01/29/22</u> | Score: _____ |

No Work ⇔ No Points

Use Pencil Only ⇔ Be Neat & Organized

1. Consider $\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ$

(a) (2 points) Use your calculator to find its exact value.

$$\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ = 1 \quad \checkmark$$

(a) _____

(b) (2 points) Use a known formula to simplify it, and then evaluate it.

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(35^\circ + 55^\circ) = \sin(90^\circ) = 1 \quad \checkmark$$

(b) _____

2. Consider $\cos 55^\circ \cos 10^\circ + \sin 55^\circ \sin 10^\circ$

(a) (2 points) Use your calculator to find its exact value.

$$\cos 55^\circ \cos 10^\circ + \sin 55^\circ \sin 10^\circ = 0.7071 \quad \checkmark$$

or $\frac{1}{\sqrt{2}}$

(a) _____

(b) (2 points) Use a known formula to simplify it, and then evaluate it.

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \cos(55^\circ - 10^\circ) \Rightarrow \cos(45^\circ) = \frac{1}{\sqrt{2}} \text{ or } 0.7071 \quad \checkmark$$

(b) _____

3. Consider $\frac{2 \tan 22.5^\circ}{1 - \tan^2 22.5^\circ}$

(a) (2 points) Use your calculator to find its exact value.

$$\frac{2 \tan 22.5^\circ}{1 - \tan^2 22.5^\circ} = 1 \quad \checkmark$$

(a) _____

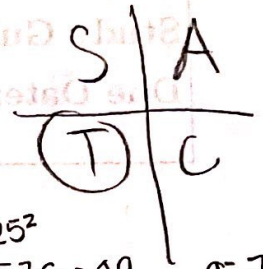
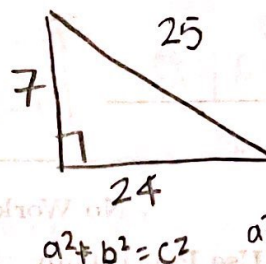
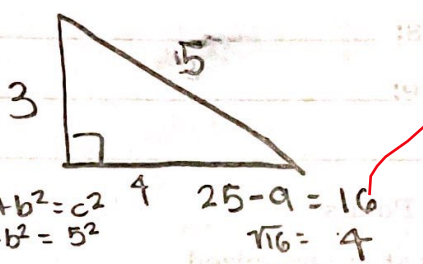
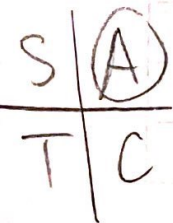
(b) (2 points) Use a known formula to simplify it, and then evaluate it.

$$\frac{2 \tan 22.5^\circ}{1 - \tan^2 22.5^\circ} = \tan(2 \cdot 22.5^\circ) = \tan 45^\circ = 1 \quad \checkmark$$

(b) _____

4. Given $\sin x = \frac{3}{5}$, $\cos y = -\frac{24}{25}$, x is in quadrant I, and y is in quadrant III.

(a) (3 points) Draw two different right triangle and clearly label them.



(b) (3 points) Find the exact value for $\sin 2x$.

$$\sin 2x = 2 \sin x \cos x$$

$$\sin 2x = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} \rightarrow \boxed{\frac{24}{25}}$$

(b) _____

(c) (3 points) Find the exact value for $\sin(x+y)$.

$$\sin(x+y) = \sin A \cos B + \cos A \sin B$$

$$= \frac{3}{5} \cdot \frac{-24}{25} + \frac{4}{5} \cdot \frac{-7}{25} = -\frac{72}{125} + -\frac{28}{125} = \frac{-100}{125} = \boxed{-\frac{4}{5}}$$

(c) _____

(d) (3 points) Find the exact value for $\cos(x-y)$.

$$\cos(x-y) = \cos A \cos B + \sin A \sin B$$

$$= \frac{4}{5} \cdot \left(\frac{-24}{25}\right) + \frac{3}{5} \cdot \left(\frac{-7}{25}\right) = \frac{-96}{125} + \frac{-21}{125} = \boxed{\frac{-117}{125}}$$

(d) _____

(e) (3 points) Find the exact value for $\tan(x+y)$.

$$\tan(x+y) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{3}{4} + \frac{7}{24}}{1 - \frac{3}{4} \cdot \frac{7}{24}} = \frac{\frac{18}{24} + \frac{7}{24}}{1 - \frac{21}{96}} = \frac{\frac{25}{24}}{\frac{75}{96}} = \frac{25}{24} \cdot \frac{96}{75} = \frac{25}{24} \cdot \frac{32 \cdot 3}{32 \cdot 3} = \frac{25}{24} \cdot \frac{32}{32} = \frac{25}{24}$$

$$\frac{25}{24} \div \frac{32}{25} = \frac{32}{24 \cdot 8} = \boxed{\frac{4}{3}}$$

(e) _____

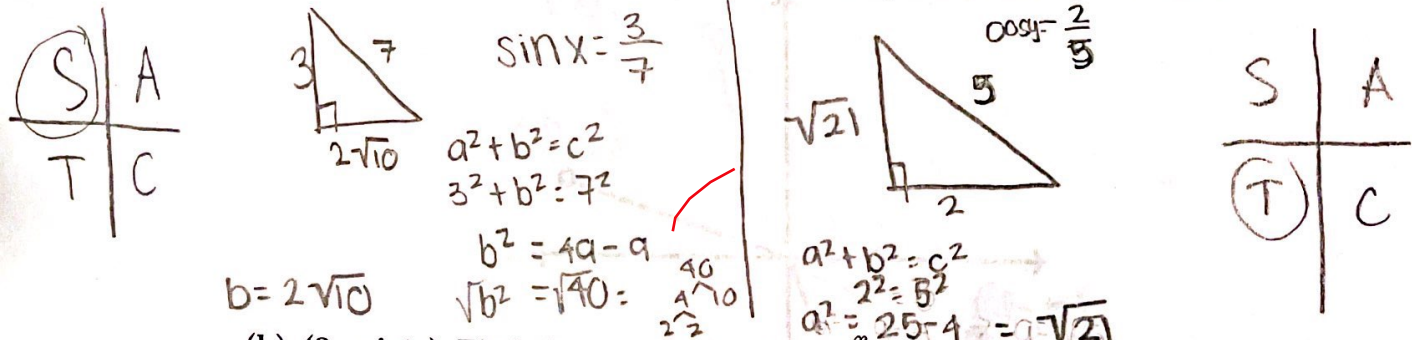
(f) (3 points) Find the exact value for $\tan \frac{y}{2}$.

$$\tan \frac{y}{2} = \frac{1 - \sin y}{1 + \cos y} \rightarrow \frac{1 - \frac{-7}{25}}{1 + \frac{-24}{25}} = \frac{\frac{32}{25}}{\frac{1}{25}} = \frac{32}{25} \cdot \frac{25}{1} = \boxed{-7}$$

(f) _____

5. Given $\sin x = \frac{3}{7}$, $\cos y = -\frac{2}{5}$, x is in quadrant II, and y is in quadrant III.

(a) (3 points) Draw two different right triangle and clearly label them.



(b) (3 points) Find the exact value for $\sin \frac{x}{2}$.

$90^\circ < x < 180^\circ$
 $45^\circ < x < 90^\circ$
 QI $\frac{x}{2}$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \Rightarrow \sqrt{\frac{1 - \frac{2\sqrt{10}}{7}}{2}} \Rightarrow \sqrt{\frac{\frac{7}{7} - \frac{2\sqrt{10}}{7}}{\frac{2}{7}}} =$$

$$\frac{\sqrt{\frac{7 - 2\sqrt{10}}{7} \cdot 2}}{2 \cdot 7} \Rightarrow \frac{\sqrt{7 - 2\sqrt{10}}}{14}$$

(c) (3 points) Find the exact value for $\cos(x - y)$.

$$\cos(x - y) = \cos A \cos B + \sin A \sin B$$

$$= \left(\frac{2\sqrt{10}}{7}\right) \left(-\frac{2}{5}\right) + \frac{3}{7} \cdot \frac{-\sqrt{21}}{5}$$

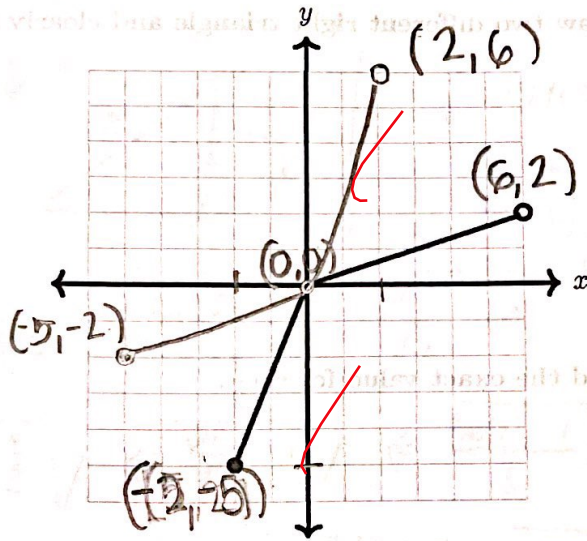
$$\frac{4\sqrt{10}}{35} + \frac{-3\sqrt{21}}{35} = \frac{4\sqrt{10} - 3\sqrt{21}}{35}$$

(d) (3 points) Find the exact value for $\tan(y - 45^\circ)$.

$$\tan(y - 45^\circ) = \tan 45^\circ = 1$$

$$\frac{\tan(\alpha - \beta)}{1 + \tan \alpha \tan \beta} = \frac{\frac{\sqrt{21}}{2} - 1}{1 + \left(\frac{\sqrt{21}}{2} \cdot 1\right)} = \frac{\frac{\sqrt{21}}{2} - \frac{2}{2}}{\frac{2}{2} + \frac{\sqrt{21}}{2}} = \frac{-\frac{2 - \sqrt{21}}{2}}{\frac{2 + \sqrt{21}}{2}} = \frac{-2 + \sqrt{21}}{2 + \sqrt{21}} \cdot \frac{2 - \sqrt{21}}{2 - \sqrt{21}} = \frac{-4 + 2\sqrt{21} - 2\sqrt{21} + 21}{4 - 21} = \frac{17 - 4\sqrt{21}}{-17}$$

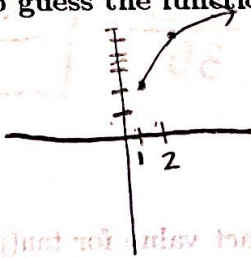
6. (5 points) Consider the graph below, draw its inverse if it exists, then complete the chart below using the interval notation.



| | Domain | Range |
|----------------------|-----------|-----------|
| Given graph | $[-5, 2)$ | $[-2, 6)$ |
| Inverse of the graph | $[-2, 6)$ | $[-5, 2)$ |

7. (3 points) Use the table below to guess the function,

| | | | | | | |
|--------|---|---|----|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| $f(x)$ | 2 | 5 | 10 | 17 | 26 | 37 |



~~$f(x) = \sqrt{x}$~~

then complete the table below and guess the inverse function.

~~$f^{-1}(x) = x^2$~~

| | | | | | | |
|-------------|---|---|----|----|----|----|
| x | 2 | 5 | 10 | 17 | 26 | 37 |
| $f^{-1}(x)$ | 1 | 2 | 3 | 4 | 5 | 6 |