Solve: $|3x - 2| < 7$

1. It is already isolated,

2. Solve $|3x - 2| = 7$
   
   $3x - 2 = 7$ or $3x - 2 = -7$
   
   $3x = 9$ or $3x = -5$
   
   $x = 3$ or $x = -\frac{5}{3}$

3. Place these ans on NL

   $(\frac{5}{3}, 3)$ and $x < 3$

S.B.N.

$\{x | -\frac{5}{3} < x < 3\}$
Solve \[ |2x + 5| \geq 3 \]

1. Make Sure Abs. Value is isolated.

2. Solve \[ |2x+5| = 3 \]
   
   \[ 2x + 5 = 3 \quad \text{or} \quad 2x + 5 = -3 \]
   
   \[ x = -1 \quad x = -4 \]

3. Place them on the N.L.\[ \infty \quad -4 \quad -1 \quad 0 \quad \infty \]

S.B.N. \[ \{ x \mid x \leq -4 \text{ or } x \geq -1 \} \]

I.N. \[ (-\infty, -4] \cup [-1, \infty) \]

---

Solve \[ 4|x+3| + 7 \geq 3 \]

1. Always isolate the abs. value.

   \[ 4|x+3| \geq 3 - 7 \]
   
   \[ 4|x+3| \geq -4 \]

   \[ |x+3| \geq -\frac{4}{4} = -1 \]

   \[ |x+3| > -1 \quad \text{All Reals} \quad \mathbb{R} \]

   \[ \text{or} \quad + \quad > -1 \quad (-\infty, \infty) \]
Solve
\[-2 | \frac{1}{2}x - 3 | - 7 > 5\]

Always isolate the Abs. Value.
\[-2 | \frac{1}{2}x - 3 | \geq 12\]
\[| \frac{1}{2}x - 3 | < \frac{12}{-2}\]
\[| \frac{1}{2}x - 3 | < -6\]
\[0, + < -6\]

NO Solution

\[\emptyset\]

\[\text{Solve } | 3x + 5 | < | x - 7 |\]

\[\text{1. Solve } | 3x + 5 | = | x - 7 |\]
\[3x + 5 = x - 7 \quad \text{or} \quad 3x + 5 = -(x - 7)\]
\[2x = -12 \quad \text{or} \quad 4x = -x + 7\]
\[\boxed{x = -6} \quad \boxed{x = \frac{1}{2}}\]

\[\text{2. Place these solutions on the number line.}\]

\[\text{S.B.N. } \left\{ x \mid -6 < x < \frac{1}{2} \right\}, \quad \text{I.N. } (-6, \frac{1}{2})\]
**February 15, 2017**

Solve \[ |2x - 3| \geq |x + 3| \]

1. Solve \[ |2x - 3| = |x + 3| \]
   
   \[
   \begin{align*}
   2x - 3 &= x + 3 \\
   2x - x &= 3 + 3 \\
   x &= 6
   
   \text{or} \\
   2x - 3 &= -(x + 3) \\
   2x - 3 &= -x - 3 \\
   2x + x &= -3 + 3 \\
   x &= 0
   \end{align*}
   \]

2. Put these on the number line:
   
   S.B.N. \{x \mid x \leq 0 \text{ or } x \geq 6 \}; I.N. \((-\infty, 0] \cup [6, \infty)\)

---

Find a system of linear inequalities that satisfies the following:

\[
\begin{align*}
\begin{cases}
  x > -4 \\
  y < 2 \\
  y \geq \frac{1}{2}x - 2
\end{cases}
\end{align*}
\]
\[ \begin{align*}
\{ x \geq -4 \\
\{ x \leq 4 \\
y > -3 \\
y \leq -\frac{3}{5}x + 3
\end{align*} \]

Square Function
\[ f(x) = x^2 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>-3</td>
<td>9</td>
</tr>
</tbody>
</table>

Diagram of square function
\[ f(x) = (x - 2)^2 + 1 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

\[ f(x) = -(x + 1)^2 + 2 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-2</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>
Graph

\[ f(x) = -\sqrt{x - 5} + 2 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>√5</td>
<td>2</td>
</tr>
<tr>
<td>√6</td>
<td>3</td>
</tr>
<tr>
<td>√9</td>
<td>4</td>
</tr>
<tr>
<td>√14</td>
<td>5</td>
</tr>
</tbody>
</table>

**1. List Perfect Squares**

- \( x - 5 = 0 \rightarrow x = 5 \)
- \( x - 5 = 1 \rightarrow x = 6 \)
- \( x - 5 = 4 \rightarrow x = 9 \)
- \( x - 5 = 9 \rightarrow x = 14 \)
- \( x - 5 = 16 \rightarrow x = 21 \)

\( x - 3 = 0 \quad x = 3 \)

\[ f(x) = -(x - 3)^2 \]
$y(x) = -\sqrt{x+3} - 4$

$x+3=0 \rightarrow x=-3$

\[
\begin{array}{c|c}
 x & y \\
 \hline
 -3 & -4 \\
 -2 & -5 \\
 1 & -6 \\
 6 & -7 \\
\end{array}
\]

\[
\begin{array}{c}
 Graph & Shade \\
\end{array}
\]

\[
\begin{align*}
 f(x) &\leq 4 \rightarrow y \leq 4 \\
 g(x) &> x^2 - 4
\end{align*}
\]
\[
\begin{align*}
\begin{cases}
    f(x) > -3 \\
    g(x) \leq -|x| + 2
\end{cases}
\end{align*}
\]

\[
\begin{array}{c|c}
 x & y \\
-2 & 0 \\
-1 & 1 \\
 0 & 2 \\
 1 & 1 \\
 2 & 0 \\
\end{array}
\]

\[
\begin{align*}
\begin{cases}
    y \geq 0 & \circlearrowleft \text{ x-axis} \\
    x \leq 4 \\
    f(x) < \sqrt{x + 4}
\end{cases}
\end{align*}
\]

\[
\begin{array}{c|c}
 x & y \\
-4 & 0 \\
-3 & 1 \\
 0 & 2 \\
 5 & 3 \\
\end{array}
\]

Solve, graph, I.N.
\[ \frac{3}{8} x + 1 > 0 \text{ OR } -2x \geq -4 \]
\[ 8 \cdot \frac{3}{8} x + 8 \cdot 1 > 8 \cdot 0 \]
\[ -2x \geq -4 \]
\[ x \leq \frac{8}{3} \text{ OR } x \leq 2 \]
\[ \text{All Reals } R \]
\[ (-\infty, \infty) \]

Solve, graph, I.N., S.B.N.
\[ -2 < -x - 12 \text{ AND } -14 < 5(x - 3) + 6x \]
\[ x < -12 + 2 \]
\[ x < -10 \text{ AND } \]
\[ x > \frac{1}{11} \]
\[ f(x) = |3x + 4| \]

1. Solve \( f(x) = 5 \)

\[ |3x + 4| = 5 \]

\[ 3x + 4 = 5 \quad \text{or} \quad 3x + 4 = -5 \]

\[ x = \frac{1}{3} \quad \text{or} \quad x = -3 \quad \{\frac{1}{3}, -3\} \]

2. Solve \( f(x) \geq 5 \)

3. Solve \( f(x) < 5 \)

\[ f(x) = \frac{\text{whatever}}{x^2 - 2x - 15} \]

\[ (x - 5)(x + 3) \]

\[ x \neq 5 \quad x \neq -3 \]

\((-\infty, -3) \cup (-3, 5) \cup (5, \infty)\)
Simplify

\[
\frac{x}{x^2 - 2x - 15} - \frac{4}{x^2 - 25}
\]

\[
= \frac{x(x + 5)}{(x-5)(x+3)(x+5)} - \frac{4(x+3)}{(x+5)(x-5)(x+3)}
\]

\[
= \frac{x(x + 5) - 4(x+3)}{(x-5)(x+3)(x+5)} = \frac{x^2 + 9x - 4x - 12}{(x-5)(x+3)(x+5)}
\]

\[
= \frac{x^2 + 5x - 12}{(x-5)(x+3)(x+5)}
\]