Solve by Cramer's rule:

\[
\begin{align*}
4x - 3y &= 10 \\
y &= x - 3
\end{align*}
\]

\[
\begin{aligned}
\Rightarrow \quad \begin{cases} 
4x - 3y &= 10 \\
-x + y &= -3
\end{cases}
\end{aligned}
\]

1. \(D = \begin{vmatrix} 4 & -3 \\ -1 & 1 \end{vmatrix} = 4(1) - (-1)(-3) = 4 - 3 = 1\)

2. \(D_x = \begin{vmatrix} 1 & -3 \\ -3 & 1 \end{vmatrix} = 10(1) - (-3)(-3) = 10 - 9 = 1\)

3. \(D_y = \begin{vmatrix} 4 & 10 \\ -1 & -3 \end{vmatrix} = 4(-3) - 10(-1) = -12 + 10 = -2\)

\[
\begin{aligned}
x &= \frac{D_x}{D} = \frac{1}{1} \\
y &= \frac{D_y}{D} = \frac{-2}{1}
\end{aligned}
\]
Solve by Cramer's Rule:
\[ \begin{align*}
6x + 5y &= 28 \\
x - 2y &= -1
\end{align*} \]

\[ D = \begin{vmatrix}
6 & 5 \\
1 & -2
\end{vmatrix} = (6(-2) - 5(1)) = -17 \]

\[ D_x = \begin{vmatrix}
28 & 5 \\
-1 & -2
\end{vmatrix} = 28(-2) - (-1)(5) = -56 + 5 = -51 \]

\[ D_y = \begin{vmatrix}
6 & 28 \\
1 & -1
\end{vmatrix} = 6(-1) - 1(28) = -34 \]

\[ x = \frac{D_x}{D} = \frac{-51}{-17} = 3 \]

\[ y = \frac{D_y}{D} = \frac{-34}{-17} = 2 \]

\[ \{ (3, 2) \} \]

---

Solve for \( z \) using Cramer's rule:
\[ \begin{align*}
x + y - z &= 0 \\
3x - y + z &= 4 \\
x + 2y - z &= 1
\end{align*} \]

\[ D = \begin{vmatrix}
1 & 1 & -1 \\
3 & -1 & 1 \\
1 & 2 & -1
\end{vmatrix} = 1 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 1(1-1) - 1(3-1) - 1(6-1) = -1 + 4 - 7 = -4 \]

\[ D_z = \begin{vmatrix}
1 & 1 & 0 \\
3 & -1 & 4 \\
1 & 2 & 1
\end{vmatrix} = 1 \begin{vmatrix} -1 & 4 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix} + 0 = -1(-1-4) - 1(3-0) = 5 \]

\[ z = \frac{D_z}{D} = \frac{-8}{-4} = 2 \]

So, \( z = 2 \).
we need 100 lb. of candy @ $3.15/lb. we have unlimited supply of candies @ $2.25/lb & $3.75/lb. How many pounds of each? use Cramer’s Rule.

\[
\begin{align*}
\begin{cases}
\ x + y &= 100 \\
100x + 3.75y &= 3150
\end{cases}
\end{align*}
\]

\[
D = \begin{vmatrix} 1 & 1 \\ 225 & 375 \end{vmatrix} = 150 \\
D_x = \begin{vmatrix} 100 & 1 \\ 3150 & 375 \end{vmatrix} = 6000 \\
D_y = \begin{vmatrix} 1 & 100 \\ 225 & 3150 \end{vmatrix} = 9000
\]

\[x = \frac{D_x}{D} = \frac{6000}{150} = 40, \quad y = \frac{D_y}{D} = \frac{9000}{150} = 60\]

Elena got $212 in simple interest from 3 different accounts in one year. Total investment was $2500. One acct. paid 7%, another one 8%, and third one 9%. She invested $1000 more in 9% account than 8% acct. How much per account?

\[
\begin{align*}
\begin{cases}
\ x + y + z &= 2500 \\
0.07x + 0.08y + 0.09z &= 212 \\
z &= y + 1000
\end{cases}
\end{align*}
\]

\[I = P \cdot R \cdot t \quad \text{For one yr, t=1} \]

\[x @ 7\%, \quad y @ 8\%, \quad z @ 9\%.\]
\[
\begin{align*}
\begin{cases}
x + y + z = 2500 \\
0.07x + 0.8y + 0.9z = 212 \\
z = y + 1100
\end{cases}
\Rightarrow
\begin{cases}
x + y + z = 2500 \\
x + 8y + 9z = 21200 \\
0x - y + z = 1100
\end{cases}
\end{align*}
\]

Solve for \( y \) only: 
\[
y = \frac{Dy}{D} = \frac{1500}{3} = \$500
\]

\[
D = \begin{vmatrix}
1 & 1 & 1 \\
7 & 8 & 9 \\
0 & -1 & 1
\end{vmatrix} = 1(8-9) - 1(7-0) + 1(-7-0) = 3
\]

\[
Dy = \begin{vmatrix}
1 & 2500 & 1 \\
7 & 21200 & 9 \\
0 & 1100 & 1
\end{vmatrix} = 1(21200+9900) - 2500(7-0) + 1(7200-0) = 1500
\]

Simplify:

\[
\begin{vmatrix}
x & y & 1 \\
0 & 3 & 1 \\
2 & 0 & 1
\end{vmatrix} = 0
\]

\[
x(3-0) - y(0-2) + 1(0-6) = 0
\]

\[
3x + 2y - 6 = 0
\]

\[
x = \text{Int} (2, 0) \\
y = \text{Int} (0, 3)
\]
Simplify
\[
\begin{vmatrix}
 x & y & 1 \\
 0 & -5 & 1 \\
 3 & 0 & 1
\end{vmatrix} = 0
\]
\[
x(-5-0) - y(0 - 3) + 1(0 - (-15)) = 0
\]
\[
-5x + 3y + 15 = 0 \quad \rightarrow \quad \text{x-intercept} (3, 0)
\]
\[
-5x + 3y = -15 \quad \rightarrow \quad \text{y-intercept} (0, -5)
\]

Solve
\[
\begin{cases}
 x^2 + y^2 = 41 \\
y = x + 1
\end{cases}
\]
\[
\begin{cases}
 (4, 5), \\
 (-5, -4)
\end{cases}
\]
\[
x^2 + (x + 1)^2 = 41
\]
\[
x^2 + (x + 1)(x + 1) = 41
\]
\[
x^2 + x^2 + x + x + 1 - 41 = 0
\]
\[
2x^2 + 2x - 40 = 0
\]
Divide by 2
\[
x^2 + x - 20 = 0
\]
\[
(x + 5)(x - 4) = 0
\]
\[
x = -5 \quad \text{or} \quad x = 4
\]
Solve
\begin{align*}
\begin{cases}
\chi^2 - y^2 = 16 \\
x + y^2 = 4
\end{cases} & \Rightarrow \\
\chi^2 + x = 20
\end{align*}
\begin{align*}
x^2 + x - 20 &= 0 \\
(x + 5)(x - 4) &= 0 \\
x &= -5, \quad x = 4
\end{align*}
\begin{align*}
\chi &= -5, \quad \chi = 4 \\
y^2 &= 9 \Rightarrow y = \pm 3 \\
\text{by inspection}
\end{align*}
\{(-5, 3), (-5, -3), (4, 0)\}

Solve
\begin{align*}
\begin{cases}
\chi^2 + y^2 = 14 \\
\chi^2 - y^2 = 4
\end{cases} & \Rightarrow \\
2 \chi^2 = 18
\end{align*}
\begin{align*}
\chi^2 &= 9 \\
\chi &= \pm 3 \\
\text{by inspection}
\end{align*}
\begin{align*}
y^2 &= 5 \\
\text{by insp.}
\end{align*}
\\frac{y = \pm \sqrt{5}}{\text{ch. II}}
\begin{align*}
\{(-3, \sqrt{5}), (3, -\sqrt{5}), (-3, \sqrt{5}), (3, -\sqrt{5})\}
Variations:

1) Directly: \( y \) varies directly as \( x \)
   
   "multiplication"
   
   \[ y = kx \]

2) Inversely: \( y \) varies inversely as \( x \)
   
   "Division"
   
   \[ y = \frac{k}{x} \]

\( k \) is the constant of variation.

ex: \( y \) varies directly as \( \text{square of } x \)

\( y \) is 50 when \( x \) is 5.

Find \( y \) when \( x \) is 10.

\[ y = 2x^2 \]

\[ \begin{align*}
50 &= k \cdot (5)^2 \\
50 &= 25k \\
k &= 2
\end{align*} \]

\[ \begin{align*}
y &= 2(10)^2 \\
y &= 200
\end{align*} \]
y varies inversely as $\text{cube of } x$

$y$ is 8 when $x$ is 2.

Find $y$ when $x$ is 4.

$y = \frac{k}{x^3}$

$8 = \frac{k}{2^3}$

$8 = \frac{k}{8}$

$k = 64$

$y = \frac{64}{4^3} = \frac{64}{64} \Rightarrow y = 1$

y varies directly as $\text{square root of } x$

$y$ is 20 when $x$ is 25.

Find $y$ when $x$ is 100.

$y = k \cdot \sqrt{x}$

$20 = k \cdot \sqrt{25}$

$20 = k \cdot 5$

$k = 4$

$y = 4 \cdot \sqrt{100} = 4 \cdot 10 = 40 \Rightarrow y = 40$
y varies inversely as the fourth power of x

y is 5 when x is 2. 

\[ y = \frac{k}{x^4} \]

Find y when x is \( \frac{1}{2} \).

\[ 5 = \frac{k}{2^4} \]

\[ y = \frac{80}{(\frac{1}{2})^4} = \frac{80}{\frac{1}{16}} = \frac{80}{\frac{1}{16}} = 1280 \]

\[ k = 80 \]

80 ÷ \( \frac{1}{16} \) = 80 \cdot \frac{16}{1} = 1280

Jointly

z varies directly as x by inversely as the square root of y.

\[ z = \frac{kx}{\sqrt{y}} \]

z is 10 when x is 2 and y is 36

Find z when x = 10 \& y = 25

\[ 10 = \frac{k \cdot 2}{\sqrt{36}} \]

\[ 10 = \frac{2k}{6} \]

\[ 60 = 2k \]

\[ k = 30 \]

\[ z = \frac{30 \cdot 10}{\sqrt{25}} \Rightarrow z = 60 \]
The electric current \( I \) varies directly as voltage \( V \).

When \( V = 15\), \( I \) is 5.

Find \( I \) when \( V \) is 50.

\[
I = \frac{1}{3} V
\]

\[
I = \frac{1}{3} (50) = \frac{50}{3}
\]

Time to do a job varies inversely as the number of people.

5 hrs for 7 people

How long if we have 35 people?

\[
T = \frac{K}{P}
\]

\[
5 = \frac{K}{7}
\]

\[
K = 35
\]

\[
T = \frac{35}{P}
\]

\[
T = \frac{35}{35} \quad 1 \text{ hr.}
\]
The intensity \( I \) of a TV signal varies inversely as the square of the distance from the TV.

\[ I = \frac{K}{d^2} \]

If \( I \) is 25 at \( d = 2 \), find \( I \) when \( d = 10 \).

\[ 25 = \frac{K}{2^2} \]

\[ 25 = \frac{K}{4} \quad \Rightarrow \quad K = 100 \]

\[ I = \frac{100}{10^2} = \frac{100}{100} = 1 \]

\[ Z \] varies directly as the square root of the sum of \( x^2 \) and \( y^2 \).

\[ Z = K \cdot \sqrt{x^2 + y^2} \]

\( Z \) is 20 when \( x \) is 3 and \( y \) is 4.

\[ 20 = k \sqrt{3^2 + 4^2} \]

\[ 20 = k \sqrt{25} \]

Find \( Z \) when \( x \) is 4 and \( y \) is 3.

\[ Z = 4 \sqrt{4^2 + 3^2} \]

\[ Z = 4 \sqrt{25} \quad \Rightarrow \quad Z = 20 \]
$Z$ varies inversely as square root of the difference of $x^2$ and $y^2$. \[ Z = \frac{K}{\sqrt{x^2-y^2}} \]

$Z$ is 8 when $x$ is 5 and $y$ is 3.

\[ 8 = \frac{K}{\sqrt{5^2-3^2}} \Rightarrow 8 = \frac{K}{4} \quad \text{K} = 32 \]

Find $Z$ when $x$ is 10 and $y$ is 6.

\[ Z = \frac{32}{\sqrt{10^2-6^2}} = \frac{32}{\sqrt{64}} = \frac{32}{8} = 4 \Rightarrow Z = 4 \]

The drag force $F$ on a boat varies directly as the surface area $A$ and square of the velocity $V$.

If velocity is 6.5 and drag force of 86 with surface area of 41.2, find the surface area when velocity is 8.2 and drag force of 92.

\[ F = K \cdot A \cdot V^2 \]

\[ 86 = K \cdot 41.2 \cdot (6.5)^2 \]

\[ K \approx 0.05 \]

\[ F = 0.05 \cdot A \cdot V^2 \]

\[ 92 = 0.05 \cdot A \cdot (8.2)^2 \]

\[ A = \frac{92}{0.05 \cdot (8.2)^2} \]

\[ A \approx 27.4 \]
Volume of a given mass of a gas varies directly as temperature and inversely as pressure. 

\[ V = \frac{kT}{p} \]

\[ 231 = \frac{k \cdot 300}{20} \]

\[ k = 15.4 \]

\[ V \text{ is 231 when } T = 300 \text{ & } P = 20. \]

Find \( V \) when \( T = 320 \) & \( P = 16. \)

\[ V = \frac{15.4(320)}{16} \]

\[ V = 308 \]

SG 11 will be posted 7-10 by Monday 11 on Tuesday.