\[ f(x) = x^2 - 4 \quad \text{and} \quad g(x) = x^2 - 9 \]

Find

1) \( (f + g)(x) \)
\[ = f(x) + g(x) \]
\[ = x^2 - 4 + x^2 - 9 \]
\[ = 2x^2 - 13 \]

2) \( (f - g)(x) \)
\[ = f(x) - g(x) \]
\[ = x^2 - 4 - (x^2 - 9) \]
\[ = x^2 - 4 - x^2 + 9 \]
\[ = 5 \]

3) \( (f \cdot g)(x) \)
\[ = f(x) \cdot g(x) \]
\[ = (x^2 - 4)(x^2 - 9) \]
\[ = x^4 - 9x^2 - 4x^2 + 36 \]
\[ = x^4 - 13x^2 + 36 \]
4) \( \frac{f(x)}{g(x)} \)

\[ = \frac{\frac{x-5}{x+2}}{\frac{x^2-9}{x^2-9}} \]

\[ = \frac{x-5}{x+2}; \quad x \neq -3, 3 \]

Domain: All Reals except ±3.

5) \( \frac{g(x)}{f(x)} \)

\[ = \frac{\frac{x^2-9}{x^2-9}}{\frac{x^2-9}{x^2-9}} \]

\[ = \frac{x^2-9}{x^2-9}; \quad x \neq ±2 \]

Domain:
All reals except ±2.

\[ f(x) = \begin{cases} \frac{x-5}{x+2} & \text{if } x \neq -2 \\ 10 & \text{if } x = -2 \end{cases} \]

\[ \text{Find} \]
1) \( f(0) = \frac{0-5}{0+2} = \frac{-5}{2} \)
2) \( f(-5) = \frac{-5-5}{-5+2} = \frac{-10}{-3} = \frac{10}{3} \)
3) \( f(-2) = 10 \)
4) \( f(5) = \frac{5-5}{5+2} = \frac{0}{7} = 0 \)
Find
1) \( A \cup B = \{a, b, c, d, e, f\} \)
2) \( A \cap B = \{c, d\} \)
3) \( A \cup C = \{a, b, c, d, x, y, z\} \)
4) \( B \cap C = \emptyset \)

\( \{x \mid x < -3\} = (-\infty, -3) \)
\( \{x \mid x \geq 4\} = [4, \infty) \)

Find
1) \( A \cup B \)
2) \( A \cap B \) (Shaded segments did not overlap. Ans. empty set.

Do not use \( \{\emptyset\} \) for empty set.
Solve

1. \(-5x + 8 \leq 3x + 32\)
   \[-5x - 3x \leq 32 - 8\]
   \[-8x \leq 24\]
   \[-\frac{8}{8}x \geq \frac{24}{-8}\]
   \[x \geq -3\]
   1) S.B.N. \(\{x \mid x \geq -3\}\)

2. \(-7 < -2x + 1 \leq 15\)
   \[-7 - 1 < -2x + 1 - 1 \leq 15 - 1\]
   \[-8 < -2x \leq 14\]
   \[-\frac{8}{-2} > -\frac{2x}{-2} \geq \frac{14}{-2}\]
   \[4 > x \geq -7\]
   2) S.B.N. \(\{x \mid -7 \leq x < 4\}\)

3) I.N. \([-7, 4)\)

4) Graph

Solve

\(-2x - 7 > 5\)
\(-2x > 12\)
\[x < -6\]

\(4x + 3 \geq 43\)
\[4x \geq 40\]
\[x \geq 10\]

\(-\infty\) \(-6\) \(10\) \(8\)

\(\{x \mid x < -6\text{ or } x > 10\}\), \((-\infty, -6) \cup (10, \infty)\)

If or was AND in the above problem,
the answer would have been \(\emptyset\)
Solve
\[ 2(x-1) \leq 4x + 6 \quad \text{AND} \quad 5x - 3 < 12 \]
\[ 2x - 2 \leq 4x + 6 \]
\[ 2x - 4x \leq 6 + 2 \]
\[ -2x \leq 8 \]
\[ x \geq -4 \]
\[ 5x < 15 \]
only the Common Shaded region.
\[ x < 3 \]

O.S.B.N. \[ \{x \mid -4 \leq x < 3\} \]

2 \[ [-4,3) \cap (-\infty,3) \]
\[ x < 3 \text{ and } x \geq -4 \]

---

Absolute Value Equations

1. Type I: \[ |x| = k \quad ; \quad k \geq 0 \]
   Ans. \( x = k \) or \( x = -k \)
   If \( k < 0 \) \( \Rightarrow \) NO Solution

Solve
\[ |2x-1| = 5 \]
\[ 2x-1 = 5 \quad \text{or} \quad 2x-1 = -5 \]
\[ 2x = 6 \quad 2x = -4 \]
\[ x = 3 \quad x = -2 \]
Final Ans \( \{3, -2\} \)
Solve

\[ 3|5x + 4| - 1 = 5 \]

\[ 3|5x + 4| = 6 \]

\[ |5x + 4| = \frac{6}{3} \Rightarrow |x| = \frac{2}{5} \]

\[ 5x + 4 = 2 \] or \[ 5x + 4 = -2 \]

\[ 5x = -2 \]

\[ x = \frac{-2}{5} \]

\[ 5x = -6 \]

\[ x = \frac{-6}{5} \]

\[ \{ \frac{-2}{5}, \frac{-6}{5} \} \]

order does not matter

Always Isolate the Abs. Value.

Solve

\[ -2|3x - 7| + 3 = 7 \]

Always Isolate abs. Value First.

\[ -2|3x - 7| = 7 - 3 \]

\[ -2|3x - 7| = 4 \]

\[ \frac{-2}{2} \left| 3x - 7 \right| = \frac{4}{2} \]

\[ \left| 3x - 7 \right| = -2 \]

\[ \emptyset = \{ \} \]
2) Type II: \[ |ax + b| = |cx + d| \]

Solve: \( ax + b = \pm (cx + d) \)

Solve \( |2x + 5| = |x - 3| \)

\[ 2x + 5 = \pm (x - 3) \]

\[ 2x + 5 = x - 3 \quad \text{or} \quad 2x + 5 = -(x - 3) \]

\[ 2x - x = -3 - 5 \]

\[ x = -8 \]

\[ \{ -8, \frac{2}{3} \} \]

\[ 3x = -2 \]

\[ x = \frac{-2}{3} \]

---

Solve \( |x - 7| = |x + 9| \)

\[ x - 7 = \pm (x + 9) \]

\[ x - 7 = x + 9 \quad \text{or} \quad x - 7 = -(x + 9) \]

\[ x - x = 9 + 7 \]

\[ 0 = 16 \implies \text{false} \]

No Solution

\[ \{ -1 \} \]
Absolute Value inequalities

|x| < 0   No Solution
|x| ≥ 0   All Real numbers

Keep in mind |x| cannot be negative and |x| is always greater than or equal to Zero.

How to Solve |ax+b| < K , |ax+b| ≤ K
|ax+b| > K   |ax+b| ≥ K

where K > 0

1. Isolate abs. value
2. Solve |ax+b| = K
3. Place your answers on the number line system
4. Shade in between for < or ≤. Shade outside for > or ≥.
5. Express your ans in other forms such as S.B.N. or I.N.
$$|2x-1| < 7$$

1) It is already isolated,

2) Solve $|2x-1| = 7$
   
   $2x-1 = 7$ or $2x-1 = -7$
   
   $2x = 8$     $2x = -6$
   
   $x = 4$     $x = -3$

3) Place these on the number line system

   ![Number line diagram]

4) Decide where to shade.  
5) Final Ans.
   
   S.B.N. $\{x \mid 3 \leq 4\}$
   I.N. $(-3, 4)$

---

$$|3x + 4| \geq 1$$

1) Abs. Value is already isolated.

2) Solve $|3x+4| = 1$

   $3x+4 = 1$ or $3x+4 = -1$

   $3x = 1 - 4$     $3x = -1 - 4$

   $3x = -3$     $3x = -5$

   $x = -1$     $x = \frac{-5}{3}$

3) Place these ans on the number line system
4. Decide where to shade.

5. Final ans.
   - S.B.N. \( \{x \mid x \leq -\frac{5}{3} \text{ or } x \geq -1\} \)
   - I.N. \((-\infty, -\frac{5}{3}] \cup [-1, \infty)\)

Solve

\[2|x+4|-1 \geq 5\]
\[\Rightarrow 1\]
\[|x+4| \geq 3\]

Now solve \(|x+4| = 3\)
\[\Rightarrow 3\]
\[x = -1, x = -7\]

Place on N.L.S.

Shade

- S.B.N. \( \{x \mid x \leq -7 \text{ or } x \geq -1\} \)
- I.N. \((-\infty, -7] \cup [-1, \infty)\)
Solve \(-3|4x-1|+2 > -13\)

To isolate Abs. Value, subtract 2, then divide by \(-3\).

\[|4x-1| < 5\]

Now solve \(|4x-1| = 5\)

\[4x-1 = 5 \quad \text{or} \quad 4x-1 = -5\]

\[4x = 6 \quad \text{or} \quad 4x = -4\]

\[x = \frac{3}{2} \quad \text{or} \quad x = -1\]

Place on the N.L.S.

\[\text{Final Ans.}\]

S.B.N. \(\{x | -1 < x < \frac{3}{2}\}\)

I.N. \((-1, \frac{3}{2})\)
Solve
\[ |x-1| \leq 4 \quad \text{AND} \quad |x+3| \geq 6 \]

\[ |x-1| = 4 \]
\[ x-1 = 4 \text{ or } x-1 = -4 \]
\[ x = 5 \text{ or } x = -3 \]

Shade between

Shade outside

\[ \{x | 3 < x \leq 5 \} \]

Final Ans

Shade the solution

\[ f(x) \leq |x-2| + 1 \]

1) \[ f(x) = |x-2| + 1 \]

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Due Tomorrow:
Project II

Connecting topics.

Try (0,0)

0 \leq |0-2| + 1

0 \leq 3 \text{ True}

3 \leq |2-2| + 1

False 3 \leq 1