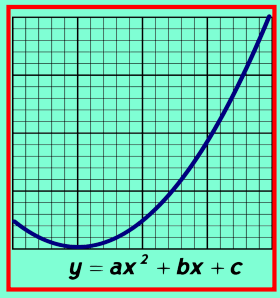
Math 125

Fall 2021

Lecture 15



Solve $|5x-3|+2=10$

isolate the abs. value

$|5x-3|=8$

Now Solve

$5x-3=8$ OR $5x-3=-8$

$5x=11$ $5x=-5$

$x=\frac{11}{5}$ $x=-1$

Solve → Solution Set $\left\{-1, \frac{11}{5}\right\}$

Solve $|4x+3| = |2x-7|$

Solve

$4x+3 = 2x-7$ OR $4x+3 = -(2x-7)$

$4x-2x = -7-3$ $\quad$ $4x+3 = -2x+7$

$2x = -10$ $\quad$ $4x+2x = 7-3$

$\boxed{x=-5}$ $\quad$ $6x = 4$

$x = \frac{4}{6}$ $\quad$ $\boxed{x=\frac{2}{3}}$

Solution set $\left\{-5, \frac{2}{3}\right\}$

Abs. Value inequality: $c>0$

$|ax+b| < c$ $\quad$ $|ax+b| \leq c$

$|ax+b| > c$ $\quad$ $|ax+b| \geq c$

1) Solve $|ax+b| = c$

2) Place Answers on the number line System

3) Shade between for $|ax+b| < c$, $|ax+b| \leq c$

4) Shade outside for $|ax+b| > c$, $|ax+b| \geq c$

Always be aware of endpoints

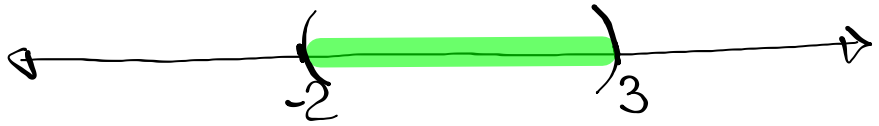
Solve $|2x-1| \leq 5$ ← shade between

Solve $|2x-1| = 5$

$2x-1=5$ OR $2x-1=-5$

$2x=6$ $\quad$ $2x=-4$

$x=3$ $\quad$ $x=-2$



S.B.N. $\{x \mid -2<x<3\}$ I.N. $(-2,3)$

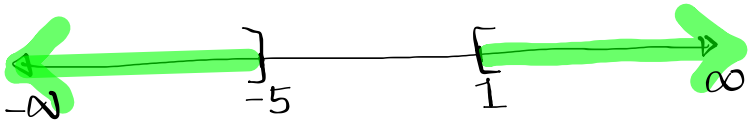
Solve $|4x+8| \geq 12$ ← shade outside

Solve $|4x+8| = 12$

$4x+8=12$ $\quad$ $4x+8=-12$

$4x=4$ $\quad$ $4x=-20$

$\boxed{x=1}$ $\quad$ $\boxed{x=-5}$



S.B.N. $\{x \mid x \leq -5$ OR $x \geq 1\}$ I.N. $(-\infty,-5] \cup [1,\infty)$

Solve $|x+4|-2 \leq 2$

Always isolate abs. value

$|x+4| \leq 4$ — Shade between

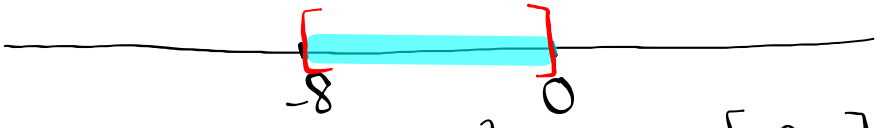
Solve $|x+4| = 4$

$x+4=4$

$x=0$

$x+4=-4$

$x=-8$



S.B.N. $\{x \mid -8 \leq x \leq 0\}$ I.N. $[-8, 0]$

Solve $|x-3|+2 > 5$

Always isolate abs. value

$|x-3| > 3$ — Shade outside

Now Solve $|x-3| = 3$

$x-3=3$

$x=6$

$x-3=-3$

$x=0$

S.B.N. $\{x \mid x<0 \text{ OR } x>6\}$

I.N. $(-\infty, 0) \cup (6, \infty)$

Solve $-3 < -2x + 5 \leq 9$

Isolate x in the middle.

$$-3 - 5 < -2x \leq 9 - 5$$

$$-8 < -2x \leq 4$$

$$\frac{-8}{-2} > \frac{-2}{-2}x \geq \frac{4}{-2}$$

$$4 > x \geq -2 \Rightarrow -2 \leq x < 4$$

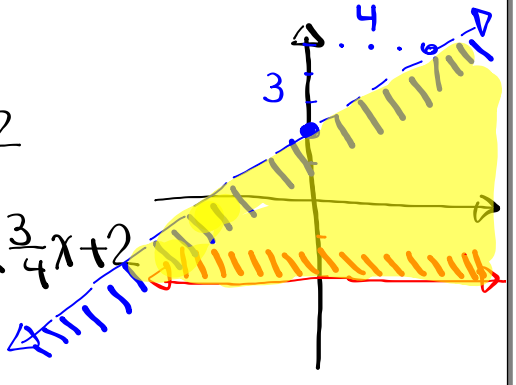
S.B.N. $\{x \mid -2 \leq x < 4\}$

I.N. $[-2, 4)$

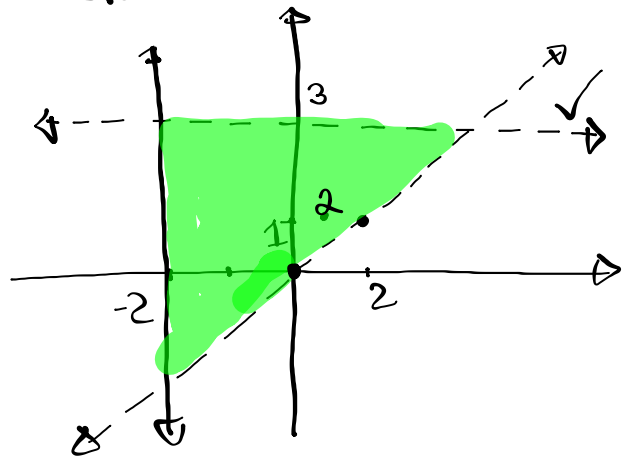
Graph

Graph & shade

$$\begin{cases} f(x) \geq -2 \\ g(x) < \frac{3}{4}x + 2 \end{cases} \Rightarrow \begin{cases} y \geq -2 \\ y < \frac{3}{4}x + 2 \end{cases}$$

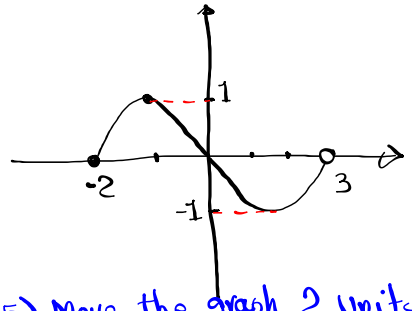


Consider the shaded region below:



$$\begin{cases} y < 3 \\ x \geq -2 \\ y > \frac{1}{2}x \end{cases}$$

Consider the graph below



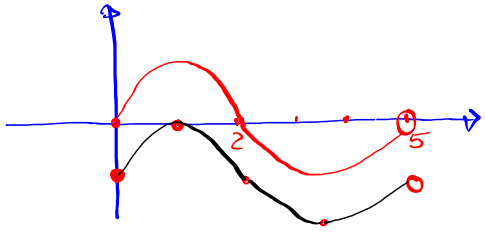
1) Domain in interval notation
[-2, 3)

2) Range in terval notation [-1, 1]

3) All intercepts
x-Ints: (-2, 0), (0, 0)
y-Int: (0, 0)

4) Function or not?
Yes, by V.L.T.

5) Move the graph 2 units to the right, then 1 unit down.



$$f(x) = \begin{cases} -\frac{3}{4}x - 3 & \text{if } x < 0 \\ \frac{1}{4}x^2 - 1 & \text{if } x \geq 0 \end{cases}$$

Find

1) $f(-4) = \frac{-3}{4}(-4) - 3$

$= 3 - 3$

$= \boxed{0}$

$\emptyset$ Not Zero

2) $f(2) = \frac{1}{4}(2)^2 - 1$

$= \frac{1}{4} \cdot 4 - 1$

$= 1 - 1 = \boxed{0}$

$$f(x) = \frac{x^2 - 16}{x^2 - 4}$$

Find

$f(-4) = \frac{(-4)^2 - 16}{(-4)^2 - 4} = \frac{16-16}{16-4}$

$= \frac{0}{12} = \boxed{0}$

$f(-2) = \frac{(-2)^2 - 16}{(-2)^2 - 4} = \frac{4-16}{4-4}$

$= \frac{-12}{0}$ $\emptyset$

undefined

Domain $x^2 - 4 \neq 0$

$x^2 \neq 4$ $x \neq \pm 2$

$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

$f(x) = 2x + 5$ $\quad g(x) = 2x - 5$

Find

1) $(f+g)(x)$

$= f(x) + g(x)$

$= 2x + 5 + 2x - 5 = \boxed{4x}$

2) $(f-g)(x)$

$= f(x) - g(x)$

$= 2x + 5 - (2x - 5)$

$= 2x + 5 - 2x + 5$

$= \boxed{10}$

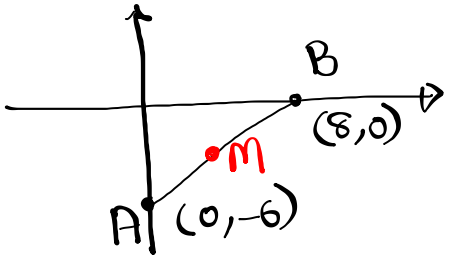
3) $(f \cdot g)(x) =$

$f(x) \cdot g(x) = (2x+5)(2x-5)$

$= 4x^2 - 10x + 10x - 25 = \boxed{4x^2 - 25}$

Given $A(0,-6)$, $B(8,0)$

Draw $\overline{AB}$



Find its midpoint

$M\left(\frac{0+8}{2}, \frac{-6+0}{2}\right) = M(4,-3)$

Find its slope

$m = \frac{-6-0}{0-8} = \frac{-6}{-8} = \boxed{\frac{3}{4}}$

Find its length.

$d = \sqrt{(0-8)^2 + (-6-0)^2}$

$= \sqrt{(-8)^2 + (-6)^2} = \sqrt{64+36}$

$= \sqrt{100} = \boxed{10}$

Factor Completely

1) $4x - 20 = 4x - 4 \cdot 5 = \boxed{4(x-5)}$

2) $x^2 - 10x = x \cdot x - 10 \cdot x = \boxed{x(x-10)}$

3) $x^3 - 100x = x\ x\ x - 100x$

$= x(x^2 - 100) = \boxed{x(x+10)(x-10)}$

Class QZ 11

Solve $|5x+10| = 20$

$5x+10 = 20$ OR $5x+10 = -20$

$5x = 10$ $5x = -30$

$\boxed{x=2}$ $\boxed{x=-6}$

$\{-6, 2\}$