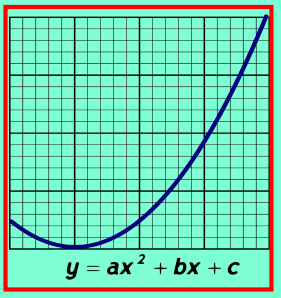
Math 125

Fall 2021

Lecture 14



Absolute Value Equations

If $|ax+b|=c$,

No solution when $c<0$

Otherwise Solve $ax+b=c$ or $ax+b=-c$

Final answer in Solution Set.

Ex: Solve $|2x-3|=9$

$2x-3=9$ OR $2x-3=-9$

$2x=12$ $2x=-6$

$x=6$ $x=-3$

$\{-3,6\}$

If we had $|2x-3|=-9 \Rightarrow$ No Solution $\emptyset$

Solve $|3x+4|=5$

$3x+4=5$ OR $3x+4=-5$

$3x=1$ $3x=-9$

$\boxed{x=\frac{1}{3}}$ $\boxed{x=-3}$

$\left\{-3, \frac{1}{3}\right\}$

If we had $|3x+4|=-5 \Rightarrow$ No Solution $\emptyset$

Solve $2|x+6|-1=19$

Hint: Isolate the Abs. Value.

$2|x+6|=20$

$|x+6|=10$

$x+6=10$ OR $x+6=-10$

$x=4$ $x=-16$

$\{-16, 4\}$

After isolating the Abs. Value if $|x+6|=-10 \Rightarrow \emptyset$

Solve $-3|2x-8|+12=15$

Hint: As always, Isolate the abs. value first.

$-3|2x-8|=15-12$

$-3|2x-8|=3$

Divide by -3

$\frac{-3}{-3}|2x-8|=\frac{3}{-3}$

$|2x-8|=-1 \Rightarrow \emptyset$ NO solution

Solve abs. value equation in the form of

$|ax+b|=|cx+d|$

Solve $ax+b=cx+d$ OR $ax+b=-(cx+d)$

Ex: Solve $|3x-7|=|x+5|$

$3x-7=x+5$ OR $3x-7=-(x+5)$

$3x-x=5+7$

$2x=12$

$\boxed{x=6}$

$3x-7=-x-5$

$3x+x=-5+7$

$4x=2$

$x=\frac{2}{4}$ $\boxed{x=\frac{1}{2}}$

$\left\{\frac{1}{2}, 6\right\}$

Solve $|2x-6| = |x+12|$

$2x-6 = x+12$ OR $2x-6 = -(x+12)$

$2x-x = 12+6$

$\boxed{x = 18}$

$2x-6 = -x-12$

$2x+x = -12+6$

$3x = -6$

$x = \frac{-6}{3}$ $\boxed{x=-2}$

$\{-2, 18\}$

Solve $|x-10| = |x+10|$

$x-10 = x+10$ OR $x-10 = -(x+10)$

$x-x = 10+10$

$0 = 20$

False

No Solution

$x-10 = -x-10$

$x+x = -10+10$

$2x = 0$

$x = \frac{0}{2}$ $\boxed{x=0}$

$\{0\}$

Solve

$|3x+1| - |2x-5| = 0$

$|3x+1| = |2x-5|$

Hint: write in the form of $|ax+b| = |cx+d|$

$3x+1 = 2x-5$ OR $3x+1 = -(2x-5)$

$3x-2x = -5-1$

$\boxed{x=-6}$

$3x+1 = -2x+5$

$3x+2x = 5-1$

$5x = 4$

$\boxed{x=\frac{4}{5}}$

$\left\{-6, \frac{4}{5}\right\}$

$A = \{x \mid x \geq -4\}$ $\quad [-4, \infty)$

$B = \{x \mid x < 5\}$ $\quad (-\infty, 5)$

Draw $A \cup B$

$(-\infty, \infty)$

All Real numbers

$\mathbb{R}$

Draw $A \cap B$

overlap

Common

$[-4, 5)$

Given $A = \{1, 2, 3, 4, 5\}$

$B = \{6, 7, 8, 9\}$

$C = \{4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A \cap B = \{\phi\} = \{\ \}$

$\{\phi\}$ WRONG

$A \cap C = \{4, 5\}$

$B \cap C = \{6\}$

Solve $3 < 2x - 3 \leq 7$

express final answer in S.B.N., interval notation, and graphing.

$3 + 3 < 2x \leq 7 + 3$

$6 < 2x \leq 10$

$\frac{6}{2} < x \leq \frac{10}{2}$

$3 < x \leq 5$

1) S.B.N. $\{x \mid 3 < x \leq 5\}$

2) Interval notation $(3, 5]$

$f(x) = x - 5$ $\quad$ $g(x) = x + 5$

Find

1) $(f+g)(x)$

$= x - 5 + x + 5 = \boxed{2x}$

2) $(f-g)(x)$

$= x - 5 - (x+5)$

$= x - 5 - x - 5 = \boxed{-10}$

3) $(f \cdot g)(x) = (x-5)(x+5)$

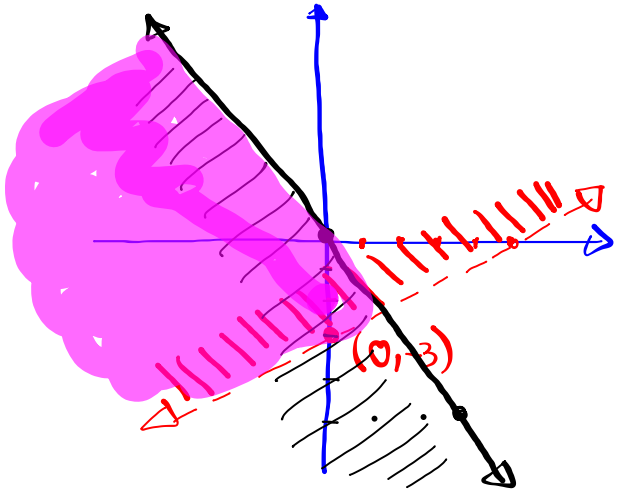
$= \boxed{x^2 - 25}$

4) $(f/g)(x)$

$= \frac{x-5}{x+5}$; $x + 5 \neq 0$, $x \neq -5$

Graph & shade

$$\begin{cases} f(x) > \frac{3}{5}x - 3 \\ g(x) \leq -\frac{5}{3}x \end{cases}$$



Zero slope $\leftrightarrow$ Horizontal line $\leftrightarrow$ $y = b$

No slope $\leftrightarrow$ Vertical line $\leftrightarrow$ $x = a$

Consider the point $(-3, 5)$