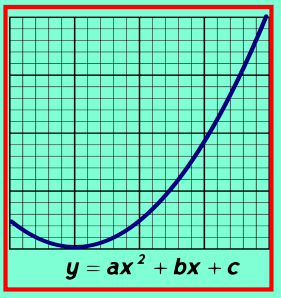
Math 125

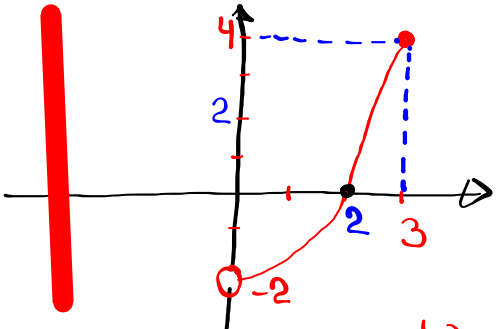
Fall 2021

Lecture 11



Class Qz 9

Consider the graph below



1) Domain
$0 < x \leq 3$ $(0, 3]$

2) Range
$-2 < y \leq 4$ $(-2, 4]$

3) All intercepts
x-Ints: $(2, 0)$
y-Ints: None

4) Function or not? Explain
Yes, by V.L.T.

$f(x) = 4x - 3$ $\quad$ $g(x) = 4x + 3$

1) $(f+g)(x) = f(x) + g(x)$

$= 4x - 3 + 4x + 3 = \boxed{8x}$

2) $(f-g)(x) = f(x) - g(x)$

$= 4x - 3 - (4x + 3)$

$= 4x - 3 - 4x - 3 = \boxed{-6}$

3) $(f \cdot g)(x) = f(x) \cdot g(x)$

$= (4x - 3)(4x + 3)$

$= 16x^2 + 12x - 12x - 9 = \boxed{16x^2 - 9}$

4) $(f/g)(x) = \dfrac{f(x)}{g(x)}$ $\quad g(x) \neq 0$ $\quad 4x + 3 \neq 0$

$4x \neq -3$

$x \neq \frac{-3}{4}$

$= \boxed{\dfrac{4x-3}{4x+3} \; ; x \neq \frac{-3}{4}}$

5) $(f \circ g)(x) = f(g(x))$

$= 4(g(x)) - 3$

$= 4(4x + 3) - 3$

$= 16x + 12 - 3 = \boxed{16x + 9}$

$$f(x) = \frac{x+8}{x-8}$$

Find

1) $f(0) = \frac{0+8}{0-8} = \frac{8}{-8} = \boxed{-1}$

2) $f(8) = \frac{8+8}{8-8} = \frac{16}{0}$ undefined

3) $f(-8) = \frac{-8+8}{-8-8} = \frac{0}{-16} = \boxed{0}$

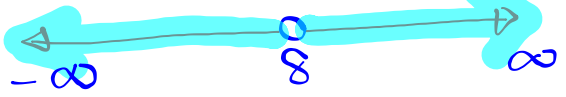
4) Discuss domain

$x - 8 \neq 0$

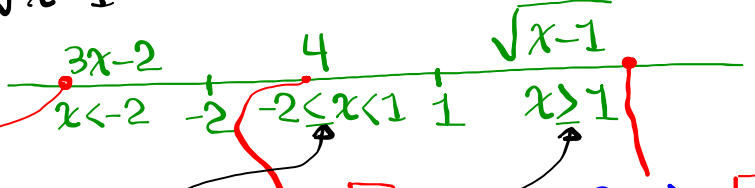
$x \neq 8$

$(-\infty, 8) \cup (8, \infty)$

OR



$$f(x) = \begin{cases} 3x-2 & \text{if } x < -2 \\ 4 & \text{if } -2 \leq x < 1 \\ \sqrt{x-1} & \text{if } x \geq 1 \end{cases}$$



$f(-3) = 3(-3)-2 = \boxed{-11}$

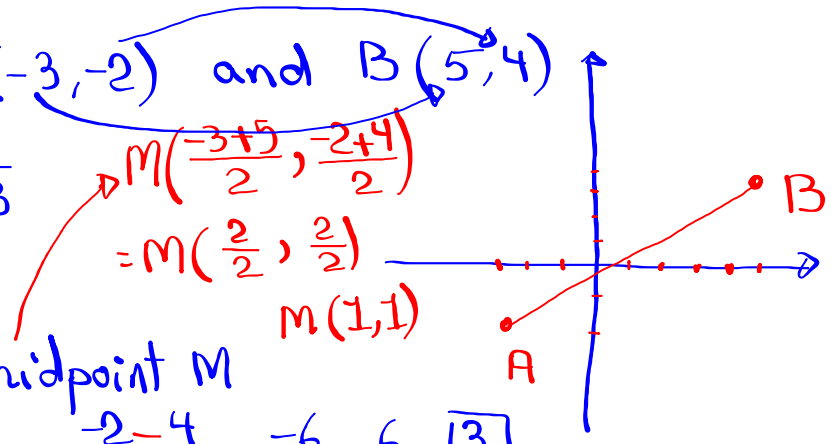
$f(-2) = \boxed{4}$

$f(0) = \boxed{4}$

$f(1) = \sqrt{1-1} = \sqrt{0} = \boxed{0}$

$f(5) = \sqrt{5-1} = \sqrt{4} = \boxed{2}$

Plot A(-3,-2) and B(5,4)



1) Draw $\overline{AB}$

2) Find midpoint M

$M\left(\frac{-3+5}{2}, \frac{-2+4}{2}\right)$

$= M\left(\frac{2}{2}, \frac{2}{2}\right)$

$M(1,1)$

3) Find slope m

$m = \frac{-2-4}{-3-5} = \frac{-6}{-8} = \frac{6}{8} = \boxed{\frac{3}{4}}$

4) Find its distance

$d = \sqrt{(-3-5)^2+(-2-4)^2}$

$= \sqrt{(-8)^2+(-6)^2}$

$= \sqrt{64+36} = \sqrt{100} = \boxed{10}$

5) Find the equation of $\overleftrightarrow{AB}$

$y = mx + b$

$y = \frac{3}{4}x + b$

use (5,4), Plug it in, and find b.

$4 = \frac{3}{4}(5) + b$

$4 = \frac{15}{4} + b$

Multiply by 4

$4 \cdot 4 = \cancel{4} \cdot \frac{15}{\cancel{4}} + 4b$

$16 = 15 + 4b$

$16 - 15 = 4b$

$1 = 4b$

$\frac{1}{4} = b$

$\boxed{y = \frac{3}{4}x + \frac{1}{4}}$

in Function notation

$\boxed{f(x) = \frac{3}{4}x + \frac{1}{4}}$

Solve, graph, give final answer in Set-Builder notation and interval notation.

$-1 < 4x + 7 \leq 19$

Subtract 7

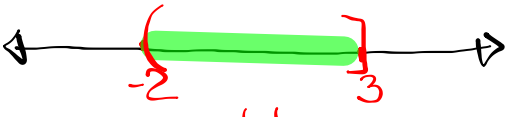
$-1-7 < 4x + 7 - 7 \leq 19 - 7$

$-8 < 4x \leq 12$

Divide by 4

$\frac{-8}{4} < \frac{4}{4}x \leq \frac{12}{4}$

$-2 < x \leq 3$



Set-Builder notation

$\{x \mid -2 < x \leq 3\}$

Interval notation

$(-2, 3]$

1) Simplify $\frac{x^4 y^2}{x y^3} = \frac{x^3}{y^1} = \boxed{\frac{x^3}{y}}$

2) Factor: $x^2 - 36 = x^2 - 6^2 = \boxed{(x-6)(x+6)}$

3) Factor $x^2 - 12x + 36 = (x-6)(x-6)$

1,36	3,12	6,6
2,18	4,9	

$= \boxed{(x-6)^2}$

Reduce $\dfrac{x^2-25}{x^2+10x+25}$

Hint: Factor Completely First

$= \dfrac{(x-5)(x+5)}{(x+5)(x+5)} = \boxed{\dfrac{x-5}{x+5}}$

Reduce $\dfrac{x^2-16}{x^2+5x-36} = \dfrac{(x-4)(x+4)}{(x-4)(x+9)} = \boxed{\dfrac{x+4}{x+9}}$

Special Factoring

$A^3 - B^3 = (A-B)(A^2+AB+B^2)$

$A^3 + B^3 = (A+B)(A^2-AB+B^2)$

$x^3-27 = x^3-3^3 = (x-3)(x^2+3x+9)$

$x^3+64 = x^3+4^3 = (x+4)(x^2-4x+16)$

$x^3-125 = x^3-5^3 = (x-5)(x^2+5x+25)$

$x^3+1000 = x^3+10^3 = (x+10)(x^2-10x+100)$

Reduce $\dfrac{x^2-4}{x^3-8} = \dfrac{x^2-2^2}{x^3-2^3}$

Hint: Factor Completely First

$= \dfrac{(x-2)(x+2)}{(x-2)(x^2+2x+4)}$

Reduce $\dfrac{x^2+14x+49}{x^3+343}$ Hint: $343 = 7^3$

$= \dfrac{x^2+14x+49}{x^3+7^3} = \dfrac{(x+7)(x+7)}{(x+7)(x^2-7x+49)}$

A^3+B^3

$= \boxed{\dfrac{x+7}{x^2-7x+49}}$

Solve $(3x-5)(2x+7)(x-8)=0$

by Zero-Factor Prop.

$3x-5=0$ $\quad 3x=5$ $\quad x=5/3$

$2x+7=0$ $\quad 2x=-7$ $\quad x=-7/2$

$x-8=0$ $\quad x=8$

$\left\{\frac{5}{3}, \frac{-7}{2}, 8\right\}$

Solve by Zero-Factor Prop:

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$x - 5 = 0$ $\qquad$ $x + 3 = 0$

$x = 5$ $\qquad$ $x = -3$

$\{-3, 5\}$

Solution Set