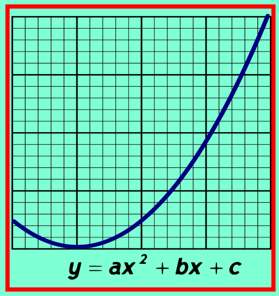
Math 125

Fall 2021

Lecture 10



Class QZ 8

1) Solve & graph

$2x - 7 \le -3x - 17$

$2x + 3x \le -17 + 7$

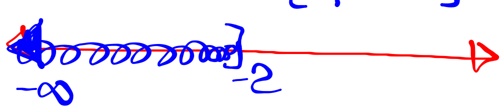
$5x \le -10$

$\frac{5}{5}x \le \frac{-10}{5}$

$\boxed{x \le -2}$

Interval Notation $(-\infty, -2]$

Set-Builder notation $\{x \mid x \le -2\}$



2) $f(x) = 5x + 2$ $\quad g(x) = 5x - 2$

Find $(f \cdot g)(x) = f(x) \cdot g(x)$

$= (5x + 2)(5x - 2)$

$= 25x^2 - \cancel{10x} + \cancel{10x} - 4 = \boxed{25x^2 - 4}$

Multiplication Product

$f(x) = 4x^2 - 6x + 10$

Find

1) $f(2) = 4(2)^2 - 6(2) + 10$
$= 4(4) - 6(2) + 10$
$= 16 - 12 + 10$
$= \boxed{14}$

2) $f(-2) = 4(-2)^2 - 6(-2) + 10$
$= 4(4) + 6(2) + 10$
$= 16 + 12 + 10$
$= \boxed{38}$

3) $f(2x^3)$
$= 4(2x^3)^2 - 6(2x^3) + 10$
$= 4(4x^6) - 6(2x^3) + 10$
$= \boxed{16x^6 - 12x^3 + 10}$

4) $f(x+3)$
$= 4(x+3)^2 - 6(x+3) + 10$
$= 4(x+3)(x+3) - 6(x+3) + 10$
$= 4(x^2 + 3x + 3x + 9) - 6(x+3) + 10$
$= 4(x^2 + 6x + 9) - 6(x+3) + 10$
$= 4x^2 + 24x + 36 - 6x - 18 + 10$
$= \boxed{4x^2 + 18x + 28}$

Given $g(x) = 2|x-4| - 8$

Find

1) $g(0)$
$= 2|0-4| - 8$
$= 2|-4| - 8$
$= 2(4) - 8$
$= 8 - 8 = \boxed{0}$

2) $g(4)$
$= 2|4-4| - 8$
$= 2|0| - 8$
$= 2(0) - 8$
$= 0 - 8 = \boxed{-8}$

3) $g(-4)$
$= 2|-4-4| - 8$
$= 2|-8| - 8$
$= 2(8) - 8 = 16 - 8 = \boxed{8}$

4) $g(x+4)$
$= 2|x\cancel{+4}\cancel{-4}| - 8$
$= \boxed{2|x| - 8}$

$f(x) = \dfrac{x-2}{x^2-4}$

Find

1) $f(0) = \dfrac{0-2}{0^2-4} = \dfrac{-2}{-4} = \boxed{\dfrac{1}{2}}$

2) $f(2) = \dfrac{2-2}{2^2-4} = \dfrac{0}{0}$

Indeterminate form

3) $f(-2) = \dfrac{-2-2}{(-2)^2-4} = \dfrac{-4}{0}$

Undefined

4) $f(3) = \dfrac{3-2}{3^2-4} = \boxed{\dfrac{1}{5}}$

$\dfrac{\text{Zero}}{\text{NonZero}} = \text{Zero}$

$\dfrac{\text{NonZero}}{\text{Zero}}$ undefined

$\dfrac{\text{Zero}}{\text{Zero}}$ Indeterminate

$f(x) = 3x+5 \qquad g(x) = 2x-5$

1) $(f+g)(x) = f(x) + g(x)$

$= 3x+5 \;\; +2x-5 \;\; = \boxed{5x}$

2) $(f-g)(x) = f(x) - g(x)$

$= 3x+5-(2x-5) = 3x+5-2x+5$

$= \boxed{x+10}$

3) $(f \cdot g)(x) = f(x) \cdot g(x)$

$= (3x+5)(2x-5)$

$= 6x^2 - 15x + 10x - 25$

$= \boxed{6x^2 - 5x - 25}$

4) $(f/g)(x) = \frac{f(x)}{g(x)}$; $g(x) \neq 0$

$= \frac{3x+5}{2x-5}$; $2x-5 \neq 0$

$2x \neq 5$

$x \neq \frac{5}{2}$

$\boxed{\frac{3x+5}{2x-5};\ x \neq \frac{5}{2}}$

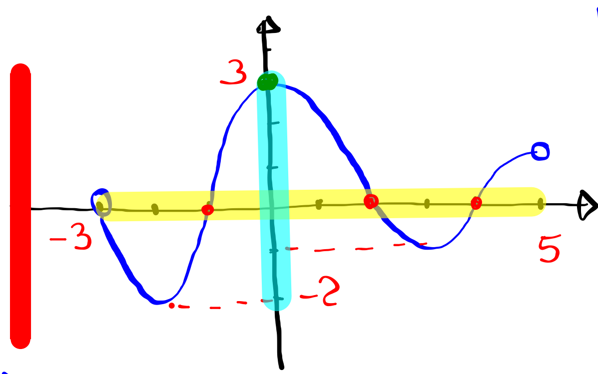
5) $(f \circ g)(x) = f(g(x))$

$= 3(g(x)) + 5$

$= 3(2x-5) + 5$

$= 6x - 15 + 5 = \boxed{6x - 10}$

Consider the graph below



1) Domain $-3 < x < 5$

$(-3, 5)$

2) Range $-2 \leq y \leq 3$

$[-2, 3]$

3) All intercepts

x-Ints: $(-1,0), (2,0), (4,0)$

Y-Int: $(0,3)$

4) Function or not? Explain

Yes, By V.L.T.

Piece-Wise Functions

$$f(x)=\begin{cases} x^2-4 & \text{if } x<0 \\ x+4 & \text{if } x\geq 0 \end{cases}$$

$f(x)=x^2-4$ $\quad$ $f(x)=x+4$

$x<0$ $\quad$ 0 $\quad$ $x\geq 0$

$f(-2)=(-2)^2-4=4-4=\boxed{0}$

$f(2)=2+4=\boxed{6}$

$f(0)=0+4=\boxed{4}$

$$f(x)=\begin{cases} |x| & \text{if } x<-2 \\ x^2 & \text{if } -2\leq x\leq 2 \\ \sqrt{x} & \text{if } x>2 \end{cases}$$

$|x|$ $\quad$ x^2 $\quad$ $\sqrt{x}$

$x<-2$ $\quad$ -2 $\quad$ $-2\leq x\leq 2$ $\quad$ 2 $\quad$ $x>2$

$f(-4)=|-4|=\boxed{4}$ ✓

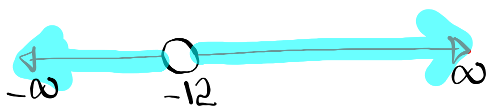
$f(0)=0^2=\boxed{0}$

$f(4)=\sqrt{4}=\boxed{2}$

September 15, 2021

$f(x) = \dfrac{x-6}{x+12}$ → x can be anything except -12

$x+12 \neq 0 \qquad x \neq -12$

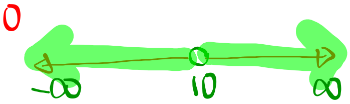


Domain $\quad (-\infty,-12) \cup (-12,\infty)$

$\uparrow$ OR

$g(x) = \dfrac{x+10}{x-10}$ → $x-10 \neq 0$

$x \neq 10$



Discuss its domain

Domain $\quad (-\infty, 10) \cup (10,\infty)$

$\uparrow$ OR

Consider the table below

x	-2	0	3	5
$f(x)$	4	1	0	-8
$g(x)$	0	3	6	2

$f(-2) = \boxed{4} \qquad g(-2) = \boxed{0}$

$(f+g)(5) = f(5) + g(5) = -8 + 2 = \boxed{-6}$

$(f-g)(0) = f(0) - g(0) = 1 - 3 = \boxed{-2}$

$(f \cdot g)(3) = f(3) \cdot g(3) = 0 \cdot 6 = \boxed{0}$

$(f/g)(-2) = \dfrac{f(-2)}{g(-2)} = \dfrac{4}{0}$ $\boxed{\text{undefined}}$

1) Simplify: $\dfrac{x^5}{x^9} = x^{5-9} = x^{-4} = \boxed{\dfrac{1}{x^4}}$

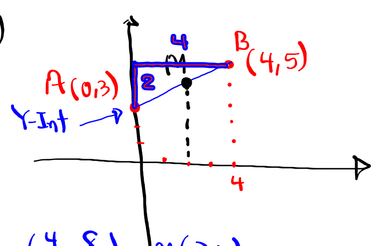
2) Simplify: $\dfrac{x^9}{x^5} = x^{9-5} = \boxed{x^4}$

3) Simplify: $\dfrac{x^2 - x - 30}{x^2 - 25} = \dfrac{(x-6)\cancel{(x+5)}}{(x-5)\cancel{(x+5)}}$

$= \boxed{\dfrac{x-6}{x-5}}$

$A(0,3)$ & $B(4,5)$

1) Draw $\overline{AB}$



2) Midpoint

$M\left(\frac{0+4}{2}, \frac{3+5}{2}\right) = M\left(\frac{4}{2}, \frac{8}{2}\right) = M(2,4)$

3) Slope $m = \frac{3-5}{0-4} = \frac{-2}{-4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

4) distance $d = \sqrt{(0-4)^2 + (3-5)^2} = \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{20} \approx \boxed{4.472}$

5) Find equation of $\overleftrightarrow{AB}$

$y = mx + b$

$\boxed{y = \frac{1}{2}x + 3}$ → $\boxed{f(x) = \frac{1}{2}x + 3}$

Slope-Int Form

Function Notation

SG 3 ✓

Looking Ahead

$f(x) = 2x - 5$

1) Replace $f(x)$ with y.

$y = 2x - 5$

2) Switch x & y.

$x = 2y - 5$

3) Now Solve for y.

$x + 5 = 2y$

$\frac{x+5}{2} = y$

4) Replace y with $f^{-1}(x)$.

$\boxed{f^{-1}(x) = \frac{x+5}{2}}$