Linear Equation in two variables:

\[ Ax + By = C \quad \text{or} \quad y = mx + b \]

\[ 3x - 2y = 10 \quad \quad \quad y = \frac{2}{5}x - 4 \]

System of linear equations in two variables

More than one equation

\[
\begin{align*}
2x - 5y &= 7 \\
x + y &= 3
\end{align*}
\]

\[
\begin{align*}
y &= -3x + 4 \\
y &= 2x - 6
\end{align*}
\]

\[
\begin{align*}
y &= \frac{2}{3}x - 8 \\
4x - 3y &= 10
\end{align*}
\]
The Solution for the system of linear equations in two variables is an ordered-pair \((x, y)\) if there is a solution.

Solution has to satisfy both equations.

Is \((3,5)\) a solution for
\[
\begin{align*}
2x + y &= 8 \checkmark \\
x + y &= 8 \\
x - y &= 2
\end{align*}
\]

So \((3,5)\) is \textbf{not} a solution.

Is \((2,-3)\) a solution of
\[
\begin{align*}
2x - y &= 7 \checkmark \\
x + 3y &= -7 \checkmark \\
2(2) - (-3) &= 7 \\
4 + 3 &= 7 \checkmark
\end{align*}
\]

So \((2,-3)\) is a solution for the system.

How to solve system of linear eqns in two variables:

1) Graphing  
2) Substitution  
3) Addition/Elimination
Solve \( \begin{cases} 3x - 4y = 12 \\ y = -\frac{2}{3}x - 3 \end{cases} \) by graphing.

System is consistent.
Eqns are independent.

Solve \( \begin{cases} 2x + 5y = 20 \\ y = -\frac{2}{5}x - 4 \end{cases} \) by graphing method.

Parallel lines
No intersections
No points in common
System is inconsistent
Eqns are independent.
Solve by graphing
\[
\begin{align*}
2x - 5y &= -20 \\
y &= \frac{2}{5}x + 4
\end{align*}
\]

infinite # of points in common \(\implies\) infinite # of solutions

System is Consistent. Eqns are dependent.

Solve \(\begin{cases} 3x - 2y = 5 \\ y = x - 4 \end{cases}\) by Substitution method.

\[3x - 2(x - 4) = 5\]
\[3x - 2x + 8 = 5\]
\[x = -3\]

\[y = \frac{-3 - 4}{3 - 4} = -7\]

Final Ans: \((-3, -7)\)

System: Consistent
Eqns: Independent
Solve by Subs. method:

\[
\begin{align*}
\begin{cases}
y &= 3x + 5 \\
6x - 2y &= -8
\end{cases}
\end{align*}
\]

\[
6x - 2(3x + 5) = -8
\]

\[
6x - 6x - 10 = -8 \implies -10 = -8
\]

System: inconsistent

Eqns: Independent

\[\emptyset\]

---

Solve by Subs. method

\[
\begin{align*}
\begin{cases}
x - 4y &= -8 \\
y &= \frac{3}{4}x + 2
\end{cases}
\end{align*}
\]

\[
3x - 4\left(\frac{3}{4}x + 2\right) = -8
\]

\[
3x - 4\cdot\frac{3}{4}x - 4\cdot2 = -8
\]

\[
3x - 3x - 8 = -8
\]

\[-8 = -8\]

\[\text{True}\]

Convex of Solutions

System: Consistent

Eqns: Dependent.
Solve by addition/elimination method:

\[
\begin{align*}
\begin{cases}
 x + y &= 5 \\
 2x - y &= 3 \\
\end{cases}
\end{align*}
\]

\[
2x = 8 \\
\Rightarrow x = 4
\]

System: Consistent

Eqns: Independent

\[
(4, 1)
\]

Solve by addition:

\[
\begin{align*}
\begin{cases}
 x + y &= -3 \\
 2x - 3y &= 19 \\
\end{cases}
\end{align*}
\]

\[
\begin{align*}
2x + y &= -3 \\
2 + y &= -3 \\
\Rightarrow y &= -5
\end{align*}
\]

\[
(2, -5) \quad \text{Consistent, Independent}
\]

\[
\begin{align*}
\begin{cases}
 3x + y &= -9 \\
 2x - 3y &= 19 \\
\end{cases}
\end{align*}
\]

\[
5x = 10 \\
\Rightarrow x = 2
\]
Solve by addition method

\[
\begin{align*}
3x + 2y &= 6 \\
4x - 3y &= 8
\end{align*}
\]

what do we do to eliminate \( y \) variable?

\[
\begin{align*}
9x + 6y &= 18 \\
8x - 6y &= 16
\end{align*}
\]

\[ \Rightarrow 17x = 34 \]
\[ \Rightarrow x = 2 \]

Final Ans

\((2, 0)\)

\[
\begin{align*}
4x - 3y &= 8 \\
4(2) - 3y &= 8 \\
8 - 3y &= 8
\end{align*}
\]

Perimeter of a rectangular garden is 36 ft.
The length is 2 ft shorter than 3 times its width. Find its dimensions

\[
2L + 2W = 36
\]

Solve

\[
\begin{align*}
2L + 2W &= 36 \\
L &= 3W - 2
\end{align*}
\]

by Subs. method

\[
2(3W-2) + 2W = 36
\]

\[
L = 13
\]

\[
W = 5
\]

5 ft by 13 ft
John has 10 coins. Nickels & Dimes only. Total value 85¢. How many of each?

\[
\begin{align*}
-5N + D &= 10 \\
5N + 10D &= 85
\end{align*}
\]

\[\Rightarrow \begin{cases} 
-5N - 5D = -50 \\
5N + 10D = 85
\end{cases} \]

\[5D = 35\]

\[D = 7\]

7 Dimes
3 Nickels

Two angles are complementary. One of them is 10° more than 3 times the other one. Find both.

\[\begin{align*}
x + y &= 90 \\
x &= 3y + 10
\end{align*}\]

\[3y + 10 + y = 90\]

\[4y = 80\]

\[y = 20\]
Two angles are supplementary. One of them is $40^\circ$ less than the other one. Find both angles.

\[
\begin{align*}
    x + y &= 180 \\
    x &= y - 40
\end{align*}
\]

\[
\begin{align*}
    y - 40 + y &= 180 \\
    2y &= 220 \\
    y &= 110
\end{align*}
\]

$110^\circ$ and $70^\circ$.

Nothing due but work on SGE 7.