System of linear equations
\[
\begin{align*}
x + y &= 5 \\
x - y &= 1
\end{align*}
\]
Solution if there is any, is an ordered pair which satisfies both equations.

Is \((3, 2)\) a solution?

\text{Eqn 1} \quad 3 + 2 = 5 \quad 5 = 5 \checkmark
\text{Eqn 2} \quad 2(3) - 2 = 1 \quad 6 - 2 = 1

\((3, 2)\) is not a Soln. \quad 4 = 1 \quad \text{False}
Is \((2,3)\) a Solution?
Eqn 1 \(2 + 3 = 5\) \(\checkmark\)  \(5 = 5\) \(\checkmark\)
Eqn 2 \(2(2) - 3 = 1\)
\(4 - 3 = 1\) \(\checkmark\)
Yes, \((2,3)\) is a Solution.

Is \((-2,4)\) a Solution of \(\begin{cases} 2x + y = 0 \\text{?} \\ 3x + y = 2 \end{cases}\)
\(2(-2) + 4 = 0\) \(\checkmark\)  \(3(-2) + 4 = 2\)
\(-4 + 4 = 0\) \(\checkmark\)  \(-6 + 4 = 2\)  \(-2 = 2\) False
\((-2,4)\) is not a Solution.

A system of linear equations could have
1) Exactly one Solution
   - System is Consistent
   - Eqns are independent
2) Infinitely many Solutions
   - System is Consistent
   - Eqns are dependent
3) No Solution at all.
   - System is inconsistent
   - Equations are independent
Solve by Graphing:
\[
\begin{align*}
x + y &= 5 \\
2x - y &= 1
\end{align*}
\]

So \( y = -2x + 1 \)

\( y = 2x - 1 \)

Solv: (2,3)

---

Solve by Graphing:
\[
\begin{align*}
3x + 2y &= 6 \\
y &= -\frac{3}{2}x - 2
\end{align*}
\]

Parallel lines

No Solution.
Solve by Graphing
\[ \begin{align*}
4x - 3y &= -6 \\
y &= \frac{4}{3}x + 2
\end{align*} \]
\[-3y = -4x - 6 \quad y = \frac{4}{3}x - \frac{6}{3} \quad y = \frac{4}{3}x + 2\]
Infinitely Many Solutions

Solve by Subs.
\[ \begin{align*}
3x + 2y &= 5 \\
y &= -x + 3
\end{align*} \]
\[3x + 2(-x + 3) = 5\]
\[3x - 2x + 6 = 5\]
\[x = 5 - 6\]
\[x = -1\]
\[y = -(-1) + 3 = 4\]
\((-1, 4)\) ordered pair
\[\{( -1, 4) \}\]
\[
\begin{align*}
2x - 3y &= 7 \quad \text{Solve by Subs. method} \\
x &= \frac{3}{2}y - 2 \\
2\left(\frac{3}{2}y - 2\right) - 3y &= 7 \\
3y - 4 - 3y &= 7 \\
-4 &= 7 \\
\text{false} \Rightarrow \emptyset
\end{align*}
\]

Solve by Subs.
\[
\begin{align*}
y &= 3x - 4 \\
6x - 2y &= 8
\end{align*}
\]
\[
\begin{align*}
6x - 2(3x - 4) &= 8 \\
6x - 6x + 8 &= 8 \\
8 &= 8 \quad \text{True} \\
\text{Infinitely Many} \\
\text{Solutions.}
\end{align*}
\]
Solve by Addition/Elimination method:

\[ \begin{align*}
\begin{cases}
  x + y &= 5 \\
 2x - y &= 1
\end{cases} \quad & 2 + y = 5 \\
& \downarrow \quad \boxed{y = 3} \quad (2, 3) \\
3x &= 6 \\
& \boxed{x = 2} \\
\end{align*} \]

\[ \begin{align*}
\begin{cases}
  3x + 2y &= 5 \\
 4x - y &= 3
\end{cases} \Rightarrow \begin{cases}
  3x + 2y &= 5 \\
 8x - 2y &= 6
\end{cases} \\
(1) + 2y &= 5 \\
\boxed{y = 1} \\
11x &= 11 \\
& \boxed{x = 1} \\
(1, 1) \quad \text{Final Ans.}
\end{align*} \]

Solve by elimination method:

\[ \begin{align*}
\begin{cases}
  3x + 5y &= -7 \\
5x + 2y &= 1
\end{cases} \Rightarrow \begin{cases}
  -6x - 10y &= 14 \\
25x + 10y &= 5
\end{cases} \\
5(1) + 2y &= 1 \\
\boxed{2y = -4} \\
\boxed{y = -2} \\
19x &= 19 \\
& \boxed{x = 1} \\
(1, -2) \quad \boxed{\{ (1, -2) \}}
\end{align*} \]
17 tkts were sold.
Adults & kids only
Adults pay $7, kids pay $3.
$79 collected. How many of each was sold?

\[
\begin{align*}
-3A + K &= 17 \\
7A + 3K &= 79
\end{align*}
\]

\[
\begin{align*}
-3A - 3K &= -51 \\
7A + 3K &= 79
\end{align*}
\]

\[
4A = 28
\]
\[
A = 7
\]

7 Adults
&
10 Kids

A rectangular billboard has a perimeter of 114 ft.
Its length is 3 ft longer than twice its width.

Find its dimensions.

\[
\begin{align*}
2L + 2W &= 114 \\
L &= 2W + 3
\end{align*}
\]

\[
2(2W + 3) + 2W = 114 \implies W = 18
\]

18 ft by 39 ft
John needs 50 liters of 18% alcohol solution. He has unlimited supply of 15% and 20% alcohol solutions. How many liters of each should he mix to obtain what he needs?

\[
\begin{align*}
\begin{cases}
15\% & \quad + \quad 20\% \\
x & \quad + \quad y = 50 \\
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\frac{15}{100}x + \frac{20}{100}y & = \frac{18}{100} \times 50 \\
\frac{15x + 20y}{100} & = \frac{18 \times 50}{100} \\
x + y & = 50 \\
5(x + 2y) & = 18 \times 50 \\
x + 2y & = 18 \times 50 \\
x + y & = 50 \\
x + 2y & = 90 \\
x & = 20 \\
y & = 30
\end{align*}
\]

20 liters of 15% alcohol and 30 liters of 20% alcohol.
Lisa is a candy store manager. She needs 100 lb. of candy at $3.20/lb. She has unlimited inventory of two types: one at $2.75/lb. and other one at $3.50/lb. How many pounds of each?

\[
\begin{align*}
\begin{cases}
2.75x + 3.50y &= 3.20(100) \\
100x + 2.75x + 3.50y &= 3.20(100)
\end{cases}
\end{align*}
\]

\[
\begin{align*}
x + y &= 100 \\
275x + 350y &= 32000
\end{align*}
\]

\[
\begin{align*}
275x - 275y &= -27500 \\
275x + 350y &= 32000 \\
75y &= 4500 \\
y &= 60
\end{align*}
\]

60 lb. of $3.50/lb. and 40 lb. of $2.75/lb.
Mike served 106 drinks. Small & large only.
Small @ $4/each. Large @ $7/each.
He collected $679. \begin{aligned} -4S + L &= 106 \\ 4S + 7L &= 679 \end{aligned}
How many of each?
\begin{aligned} -4S -4L &= -424 \\ 4S + 7L &= 679 \end{aligned}
\begin{aligned} \frac{-3L = 255}{L = 85} \quad \text{85 Large} \end{aligned}
\begin{aligned} 21 \quad \text{Small} \end{aligned}

Complementary Angles $\Rightarrow x^\circ + y^\circ = 90^\circ$

Supplementary Angles $\Rightarrow x^\circ + y^\circ = 180^\circ$

Vertical Angles $\Rightarrow x^\circ = y^\circ$

SG 7 Due Tomorrow at 6:00 AM
System of linear equations in three variables.

Final Ans: ordered-triple \((x, y, z)\)

\[
\begin{align*}
2x - 3y + 4z &= 7 \\
y + 3z &= -4 \\
z &= 1
\end{align*}
\]

\[
\begin{align*}
2x - 3(-7) + 4(1) &= 7 \\
2x + 21 + 4 &= 7 \\
2x &= -18 \\
x &= -9
\end{align*}
\]

\((-9, -7, 1)\)

\[
\begin{align*}
\begin{cases}
x - y + z = 10 \\
x - z = 4 \\
y - 2z = 5
\end{cases}
- \begin{cases}
x - y + z = 10 \\
x - z = 4
\end{cases}
\end{align*}
\]

\[
\begin{align*}
-y + 2z &= 6 \\
y - 2z &= 5
\end{align*}
\]

\(0 = 11 \Rightarrow \text{False}\)
\[
\begin{align*}
\begin{cases}
    x + y + z &= 6 \\
    2x - y + z &= 3 \\
    x + 2y - z &= 2
\end{cases}
\end{align*}
\]

Take 2 eqns & eliminate one variable:
\[
\begin{align*}
\begin{cases}
    x + y + z &= 6 \\
    2x - y + z &= 3
\end{cases} & \quad 2 \begin{cases}
    x + 2y - z &= 2 \\
    5x + z &= 8
\end{cases}
\end{align*}
\]

\[
\begin{align*}
3x + 2z &= 9 \\
5x + z &= 8
\end{align*}
\]

Take one of these 2 equation with the other equation not chosen earlier, and eliminate the same variable.

Now we Solve a system of 2 eqns &
2 unknowns
\[
\begin{align*}
\begin{cases}
    3x + 2z &= 9 \\
    5x + z &= 8
\end{cases} & \quad \Rightarrow \begin{cases}
    3x + 2z &= 9 \\
    -10x - 2z &= -16
\end{cases}
\end{align*}
\]

\[
\begin{align*}
5(1) + z &= 8 \\
\underline{z &= 3}
\end{align*}
\]

\[
\begin{align*}
x + y + z &= 6 \\
1 + y + 3 &= 6 \quad \boxed{y = 2}
\end{align*}
\]

Final Ans. (1, 2, 3)
Solve
\[
\begin{align*}
3x + 2y - z &= 5 \\
4x + y &= 1 \quad * \\
\end{align*}
\]
\[
\begin{align*}
2x + 3y - 2 &= 15 \\
-x + 2y &= 11 \quad * \\
\end{align*}
\]
\[
\begin{align*}
-x + 2y &= 11 \quad * \\
-9x + y &= 9 \\
4x + y &= 1 \\
\end{align*}
\]
\[
\begin{align*}
\frac{1}{x} - \frac{1}{x+1} &= \frac{1}{20} \\
\text{Solve:} \quad \frac{1}{x} - \frac{1}{x+1} &= \frac{1}{20} \quad \text{L.C.D. = 20x(x+1)} \\
\text{E.N.:} \quad 0, -1
\end{align*}
\]

The difference of reciprocals of two consecutive integers is \(\frac{1}{20}\). Find all such integers.

Cons. Integers \(\Rightarrow x, x+1\)
Reciprocal \(\Rightarrow \frac{1}{x} \text{ and } \frac{1}{x+1}\)
\[
\frac{1}{x} - \frac{1}{x+1} = \frac{1}{20}
\]
\[20x(x+1) \cdot \frac{1}{x} - 20x(x+1) \cdot \frac{1}{x+1} = \]
\[20x(x+1) \cdot \frac{1}{20} \]

\[20(x+1) - 20x = x(x+1) \]
\[20x + 20 - 20x = x^2 + x \]
\[x^2 + x - 20 = 0 \]
\[(x + 5)(x - 4) = 0 \]
by Z.F.T. \[x + 5 = 0 \quad x = -5 \]
\[x - 4 = 0 \quad x = 4 \]

Maria drove 90 miles in the city, and 130 miles on the highway. Her speed on the highway was 20 mph more than her speed in the city. Total time 4 hrs. Find her speed in the city & on the highway.
<table>
<thead>
<tr>
<th>Categories</th>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
<td>$x$</td>
<td>$t_{city}$</td>
<td>$90$</td>
</tr>
<tr>
<td>Highway</td>
<td>$x+20$</td>
<td>$t_{highway}$</td>
<td>$130$</td>
</tr>
</tbody>
</table>

\[
\frac{d}{r} = t
\]

\[
t_{city} + t_{highway} = 4
\]

\[
\frac{90}{x} + \frac{130}{x+20} = 4
\]

\[
\text{LCD} = x(x+20)
\]

\[
x > 0
\]

\[
90(x+20) + 130x = 4x(x+20)
\]

\[
90x + 1800 + 130x = 4x^2 + 80x
\]

\[
4x^2 + 80x - 90x - 1800 - 130x = 0
\]

\[
4x^2 - 140x - 1800 = 0
\]

Divide by 4 & reduce

\[
x^2 - 35x - 450 = 0
\]

\[
a = 1 \quad b = -35 \quad c = -450
\]
\[ b^2 - 4ac = (-35)^2 - 4 \cdot 1 \cdot (-450) \]
\[ = 1225 + 1800 \]
\[ = 3025 \]
\[ \chi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-35) \pm \sqrt{3025}}{2(1)} \]
\[ = \frac{35 \pm 55}{2} \]
\[ \chi = \frac{35 + 55}{2} \]
\[ \chi = 45 \]
\[ \chi = \frac{35 - 55}{2} \]
\[ \chi = -10 \]

45 mph in the City
65 mph on the highway

Every Friday
9:30 - 11:30
in E7 - 210

Special Tutor

Factoring: ---
( ) ( )