Quadratic Equation: $ax^2 + bx + c = 0; \ a \neq 0$

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b^2-4ac$ is called the discriminant.

$\begin{cases} 
> 0 & \text{Two real solutions} \\
= 0 & \text{One repeated real solution} \\
< 0 & \text{Two imaginary solutions}
\end{cases}$
Solve $x^2 - 6x + 10 = 0$ by completing the square.

\[ x^2 - 6x + 9 = -10 + 9 \]

\[ a = 1, \ b = -6, \ c = 10 \]

\[ b^2 - 4ac = (-6)^2 - 4(1)(10) = 36 - 40 = -4 \]

Since $b^2 - 4ac < 0$,

\[ (x - 3)^2 = -1 \]

by S.R.M.

\[ x - 3 = \pm \sqrt{-1} \]

\[ x = 3 \pm i \]

\[ \{3 \pm i\} \]

Two complex solutions.

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Solve $3x^2 - 2x = 5$ by Quadratic Formula.

\[ 3x^2 - 2x - 5 = 0 \]

\[ a = 3, \ b = -2, \ c = -5 \]

\[ b^2 - 4ac = (-2)^2 - 4(3)(-5) = 64 \]

Since $b^2 - 4ac > 0 \Rightarrow$ Two real solutions:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-(-2) \pm \sqrt{64}}{2(3)} = \frac{2 \pm 8}{6} \]

\[ x = \frac{2 + 8}{6} = \frac{10}{6} = \frac{5}{3} \]

\[ x = \frac{2 - 8}{6} = \frac{-6}{6} = \frac{-1}{1} \]

\[ \{\frac{1}{3}, \frac{5}{3}\} \]
Find the value for the discriminant for $4x^2 + 12x + 9 = 0$, discuss the type of solutions, then find the solutions.

Discriminant $\Rightarrow b^2 - 4ac = 12^2 - 4(4)(9) = 0$

$a=4$, $b=12$, $c=9$

Since $b^2 - 4ac = 0 \Rightarrow$ one repeated real Soln.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{0}}{2(4)} = \frac{-12}{8} = \frac{-3}{2}$$

$$\left\{ \frac{-3}{2} \right\}$$

( $2x + 3)(3x - 2) = 28$

1) Foil, Simplify, write in $ax^2 + bx + c = 0$ form $6x^2 - 4x + 9x - 6 - 28 = 0$

$6x^2 + 5x - 34 = 0$

2) Find the value of its discriminant, then discuss solutions. $b^2 - 4ac = 5^2 - 4(6)(-34)$

$b^2 - 4ac > 0 \Rightarrow$ two real Solns

$$= 5^2 - 4(6)(-34)$$

$$= 81$$

$$\Rightarrow \left\{ \frac{-17}{6}, \frac{2}{6} \right\}$$

3) Find all solutions.

$$x = \frac{-5 \pm \sqrt{841}}{2(6)} = \frac{-5 \pm 29}{12}$$

$$x = \frac{-5 + 29}{12} = \frac{24}{12} = 2$$

$$x = \frac{-5 - 29}{12} = \frac{-34}{12} = \frac{-17}{6}$$
\[(2x-3)^2 + 16 = -84\]

1) "foil, simplify, write in \(ax^2 + bx + c = 0\)
   
   \[(2x-3)(2x-3) + 16 + 84 = 0\]  
   \[4x^2 - 12x + 109 = 0\]  
   \[4x^2 - 12x + 9 + 100 = 0\]

2) "find the value of its discriminant."
   
   "Discus type of solutions."
   
   \[b^2 - 4ac = \]
   \[(-12)^2 - 4(4)(109) = -1600\]
   
   \[b^2 - 4ac < 0 \implies \text{Two Complex Solutions.}\]

3) "find all solutions."
   
   \[(2x-3)^2 + 16 = -84\]  
   \[\{2 \pm 5i\}\]  
   S.R.M.

\[\begin{cases} 
2x = 3 \pm 10i \\
2x = 3 \pm 5i \\
x = \frac{3}{2} \pm 5i 
\end{cases}\]

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Solve \((2x-5)^2 - 8(2x-5) + 20 = 0\)

"make a subs. \(\Rightarrow \) Let \(u = 2x-5\)"

\[u^2 - 8u + 20 = 0\]

\[u^2 - 8u + 16 = -20 + 16\]  
\[\begin{cases} 
(u-4)^2 = -4 
\end{cases}\]  
by S.R.M.

\[u = 4 \pm 2i \implies 2x - 5 = 4 \pm 2i\]

\[2x = 9 \pm 2i\]  
\[x = \frac{9}{2} \pm i\]  
\[\{\frac{9}{2} \pm i\}\]
Solve \( x^4 - 5x^2 - 36 = 0 \) by making a subs. \( u = x^2 \).

\[ u^2 = (x^2)^2 = x^4 \]

So

\[ u^2 - 5u - 36 = 0 \]

\[ (u - 9)(u + 4) = 0 \]

\[ u = 9 \quad u = -4 \]

\[ x^2 = 9 \quad x^2 = -4 \]

\[ x = \pm 3 \quad x = \pm 2i \]

\[ \{ \pm 3, \pm 2i \} \]