y varies directly as x.

\[ y = kx \]

\[ y = 11 \] when \( x = \frac{1}{3} \)

Find \( y \) when \( x = 3 \).

\[ 11 = k \cdot \frac{1}{3} \]

\[ k = 33 \]

\[ y = 33x \]

\[ y = 33(3) \]

\[ y = 99 \]
y varies inversely as x.  
\[ y = \frac{k}{x} \]

y is \( \frac{1}{8} \) when x is 16  
\[ \frac{1}{8} = \frac{k}{16} \]

Find y when x is \( \frac{1}{8} \).  
\[ y = \frac{2}{x} \]

\[ y = \frac{2}{\frac{1}{8}} \]

\[ y = 2 \cdot \frac{8}{1} = 16 \]

\[ y = 16 \]

y varies directly as cube of x.  
\[ y = k \cdot x^3 \]

y is 9 when x is 3.  
\[ 9 = k \cdot 3^3 \]

Find y when x is 6.  
\[ y = \frac{1}{3} x^3 \]

\[ y = \frac{1}{3} (6)^3 \]

\[ y = 72 \]

\[ k = \frac{9}{27} \]

\[ k = \frac{1}{3} \]
\( Z \) varies directly as \( x^5 \) and inversely as \( y \). 

\[
Z = \frac{K \cdot x^5}{y} \quad \frac{16}{4} = \frac{K \cdot 2^5}{4} \quad 16 = 8K \quad K = 2
\]

\( Z \) is 16 when \( x \) is 2 and \( y \) is 4.

Find \( Z \) when \( x \) is 4 and \( y \) is 8.

\[
Z = \frac{2 \cdot 4^5}{8} \quad Z = \frac{2 \cdot (4)^5}{8} \quad Z = 256
\]

The weight of a ball varies directly as cube of its radius. \( W = K \cdot R^3 \).

When radius is 2 inches, the ball weighs 1.2 pounds. \( 1.2 = K \cdot 2^3 \)

Find the weight of the ball if the radius is 3 inches.

\[
W = 0.15R^3 \quad W = 0.15(3)^3 \quad W = 4.05 \quad 4.05 \text{ pounds}
\]
The weight of an object on or above Earth varies inversely as square of the distance between the object and center of Earth. Assume the radius of Earth is 4000 miles. \( w = \frac{k}{D^2} \)

A person weighs 160 pounds on the surface of Earth. \( 160 = \frac{k}{4000^2} \)

Find this person weight if he/she is 200 miles above the surface of Earth.

\[
\begin{align*}
W &= \frac{2560000000}{4200^2} \\
W &= 145 
\end{align*}
\]

Volume of a right circular cone varies jointly as its height and square of its radius. \( V = khr^2 \)

If volume is \( 32\pi \) in\(^3\), radius is 4 in. and height is 6 in. \( 32\pi = k \cdot 6 \cdot 4^2 \)

Find the volume if radius is 6 in. and height is 4 in.

\[
\begin{align*}
V &= \frac{\pi r^2 h}{3} \\
V &= \frac{\pi (4)^2 \cdot 4}{3} \quad V = 48\pi \text{ in}^3
\end{align*}
\]
The number of cars manufactured on an assembly line varies jointly as the number of workers, and amount of time they work. \( C = kWT \)

200 workers produced 60 cars in 2 hrs. Find the # of cars if we have 240 workers in 3 hrs.

\[
\begin{align*}
C &= \frac{3}{20} WT \\
\frac{60}{400} &= k \\
\frac{6}{40} &= k \\
\frac{3}{20} &= k
\end{align*}
\]

\( C = 108 \) 108 Cars

Cross-Multiply, then Solve

\[
\frac{x}{x-4} = \frac{x+4}{6}
\]

\[
(x+4)(x-4) = 6x
\]

\[
x^2 - 16 = 6x
\]

\[
x^2 - 6x - 16 = 0
\]

\[
(x-8)(x+2) = 0
\]

\[
\begin{align*}
x-8 &= 0 \\
x &= 8 & x+2 &= 0 \\
x &= -2
\end{align*}
\]

\{ -2, 8 \}
Solve
\[
\frac{15}{x+4} = \frac{x-4}{x}
\]

Hint: Cross-Multiply
\[
(x+4)(x-4) = 15x
\]
\[
x^2 - 16 = 15x
\]
\[
x^2 - 15x - 16 = 0
\]
\[
(x-16)(x+1) = 0
\]
\[
x = 16, \quad x = -1
\]

\[
\{-1, 16\}
\]

Linear motion
\[
\text{distance} = \text{rate} \cdot \text{time}
\]
if
\[
\text{you drive 60 mph for 2.5 hrs,}
\]
\[
d = r \cdot t
\]
\[
d = 60(2.5)
\]
\[
d = 150
\]

150 miles
John drove 150 miles in the same time that Lisa drove 210 miles. Lisa was driving 20 mph faster than John. Find speed for both.

<table>
<thead>
<tr>
<th></th>
<th>( d )</th>
<th>( r )</th>
<th>( t )</th>
<th>( \frac{d}{r} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>150</td>
<td>( x )</td>
<td>( t )</td>
<td>( \frac{150}{x} )</td>
</tr>
<tr>
<td>Lisa</td>
<td>210</td>
<td>( x+20 )</td>
<td>( t )</td>
<td>( \frac{210}{x+20} )</td>
</tr>
</tbody>
</table>

\[
\frac{150}{x} = \frac{210}{x+20} \]

\[
210x = 150(x + 20)
\]

\[
210x = 150x + 3000
\]

\[
210x - 150x = 3000
\]

John 50 mph
Lisa 70 mph
\( x = 50 \)

Jose drove 90 miles in the city. Once he got to the freeway, he increased his speed by 20 mph, and drove for 130 miles.

He drove for a total of 4 hrs.

Find his speed on the freeway.

<table>
<thead>
<tr>
<th></th>
<th>( d )</th>
<th>( r )</th>
<th>( t )</th>
<th>( \frac{d}{r} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
<td>90</td>
<td>( x )</td>
<td>( t_1 )</td>
<td>( \frac{90}{x} )</td>
</tr>
<tr>
<td>Freeway</td>
<td>130</td>
<td>( x+20 )</td>
<td>( t_2 )</td>
<td>( \frac{130}{x+20} )</td>
</tr>
</tbody>
</table>

\[
t_1 + t_2 = 4 \]

\[
\frac{90}{x} + \frac{130}{x+20} = 4
\]
\[
\frac{90}{x} + \frac{130}{x+20} = \text{L.C.D.} = x(x+20)
\]

\[
x(x+20) \cdot \frac{90}{x} + x(x+20) \cdot \frac{130}{x+20} = x(x+20) \cdot 4
\]

\[
90(x+20) + 130x = 4x(x+20)
\]

\[
\frac{90x + 1800}{x} + \frac{130x}{x+20} = 4x^2 + 80x
\]

\[
220x + 1800 = 4x^2 + 80x
\]

\[
4x^2 + 80x - 220x - 1800 = 0
\]

\[
4x^2 - 140x - 1800 = 0 \rightarrow x = 45
\]

Divide by 4 to reduce

\[
x = 10
\]

\[
(45, 10, 65, 85)
\]

Mary drove 135 miles on the freeway. She reduced her speed by 25 mph and drove 40 miles in the mountain. Total time 5 hrs. Find her speed in the mountain.

\[
\begin{array}{c|c|c}
\text{Fwy} & 135 & x \\
\hline
\text{Mtn} & 40 & x-25
\end{array}
\]

\[
t_1 + t_2 = 5
\]

\[
\frac{135}{x} + \frac{40}{x-25} = 5
\]

\[
\text{L.C.D.} = x(x-25)
\]

\[
135(x-25) + 40x = 5x(x-25)
\]

Divide by 5 to reduce

\[
27(x-25) + 8x = x(x-25)
\]

\[
27x - 675 + 8x = x^2 - 25x
\]

\[
35x - 675 = x^2 - 25x
\]
\begin{align*}
\chi^2 - 25\chi - 35\chi + 675 &= 0 \\
\chi^2 - 60\chi + 675 &= 0 \\
(x - 45)(x - 15) &= 0 \\
\chi &= 45, \quad \chi &= 15
\end{align*}

45 mph on the FWY
20 mph in the Mountain.

\begin{align*}
\text{(14)} \quad \chi^3 - 5\chi^2 - 4\chi + 20 &= 0 \\
\text{Factor by Grouping} \\
\chi^2(\chi - 5) - 4(\chi - 5) &= 0 \\
(\chi - 5)(\chi^2 - 4) &= 0
\end{align*}

\begin{align*}
\text{#18} \quad 2\chi^3 &= 98\chi \\
2\chi^3 - 98\chi &= 0 \\
2\chi(\chi^2 - 49) &= 0
\end{align*}
24) \[ x^2 + (2x + 2)^2 = (2x + 3)^2 \]

\[ x^2 + (2x + 2)(2x + 2) = (2x + 3)(2x + 3) \]

\[ x^2 + 4x^2 + 8x + 4 = 4x^2 + 12x + 9 \]

Simplify
RhS = 0

Factor LHS

32) \((3x - 2)(2x - 3) = -1\)

Simplify

\[ 6x^2 - 13x + 6 = -1 \]

RhS = 0

\[ a = \quad b = \quad c = \]

#33 \[ 0.6x - 4x^2 + 1 = 0 \]

Multiply by 10 \[ 6x - 4x^2 + 10 = 0 \]

Order \[ -4x^2 + 6x + 10 = 0 \]

Divide by -2
#49 \( \frac{1}{x} + \frac{1}{x+1} = \frac{5}{6} \) \[ \text{L.C.D} = 6x(x+1) \]

\[ 6(x+1) + 6x = 5x(x+1) \]

\[ 6x + 6 + 6x = 5x^2 + 5x \]

\[ \frac{6}{x} + \frac{5}{x} - \frac{6}{x^2} = 0 \] \[ \text{L.C.D} = x^2 \]

\[ 6x^2 + 5x - 6 = 0 \]

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\[ \frac{1}{x} + \frac{1}{x+2} = 1 \]

\[ \frac{2.4}{x} + \frac{2.4}{x+2} = 1 \] \[ \text{L.C.D} = x(x+2) \]

\[ x^2 - 2.8x - 4.8 = 0 \]

\[ 10x^2 - 28x - 48 = 0 \]

\[ 5x^2 - 14x - 24 = 0 \]

\[ t = 2.4 \text{ hrs} \]

\[ \text{Work} = \text{How long} \times \text{How Fast} \]

\[ \text{Work} = t \times \text{Rate} \]

\[ 2.4(x+2) + 2.4x = x(x+2) \]

\[ 2.4x + 4.8 + 2.4x = x^2 + 2x \]

\[ 4.8x + 4.8 = x^2 + 2x \]

\[ x^2 + 2x - 4.8x - 4.8 = 0 \]

Cont. with Factoring or Q.D. Formula.
Class Quiz

1. Solve $3x(2x-3)=0$

   $3x=0 \quad 2x-3=0$

   $x=0 \quad 2x=3 \quad x=\frac{3}{2}$

   $\{0, \frac{3}{2}\}$

2. Solve $(x+5)(x-3)=9$

   $x^2-3x+5x-15-9=0 \quad (x+6)(x-4)=0$

   $x^2+2x-24=0 \quad x=-6 \quad x=4 \quad \{-6, 4\}$

3. Use the formula to solve $2x^2-3x+1=0$

   $a=2, \ b=-3, \ c=1$

   $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

   $b^2-4ac=(-3)^2-4(2)(1)=9-8=1$

   $x=\frac{3\pm\sqrt{1}}{2(2)}$

   $\{\frac{1}{2}, 1\}$

   $x=\frac{3+1}{4}$

   $x=\frac{3+1}{4}=\frac{4}{4}=1$

   $\frac{3+1}{4}=\frac{4}{4}=\frac{1}{2}$