Discrete Random Variables

&

Probability Distributions

What is a **Random Variable**?

It is a quantity whose values are real numbers and are determined by the number of desired outcomes of an experiment.

Example:

List the sample space of all outcomes for a family with two children, and the corresponding values of the number of boys.

Solution:

Sample Space	Number of Boys
BB	2
BG	1
GB	1
GG	0

Is there any special **Random Variables**?

We can categorize random variables in two groups:

- Discrete random variable
- Continuous random variable

What are **Discrete Random Variables**?

It is a numerical value associated with the desired outcomes and has either a finite number of values or infinitely many values but countable such as whole numbers $0, 1, 2, 3, \cdots$.

What are **Continuous Random Variables**?

It has infinitely numerical values associated with any interval on the number line system without any gaps or breaks.

Example:

Classify the following random variables as discrete or continuous.

1. The number of accidents on the 60 freeway.
2. The length of time it takes to play a baseball game.
3. The amount of milk produced by a cow in a month.
4. The number of correct answers on a multiple-choice exam.

Solution:

Classify the following random variables as discrete or continuous.

1. The number of accidents. $\Rightarrow$ Discrete
2. The length of time. $\Rightarrow$ Continuous
3. The amount of milk. $\Rightarrow$ Continuous
4. The number of correct answers. $\Rightarrow$ Discrete

What is a **Probability Distribution**?

It is a description and often given in the form of a graph, formula, or table, that provides the probability for all possible desired outcomes of the random variable.

Are there any **Requirements**?

Let x be any random variable and $P(x)$ be the probability of the random variable x, then

- $\sum P(x) = 1$
- $0 \leq P(x) \leq 1$

Example:

Let x be the number of defective items when 2 items have shipped and $P(x)$ be probability of x defective items. Use the table below to find $P(x = 2)$.

x	0	1	2
$P(x)$	$\frac{2}{5}$	$\frac{8}{15}$	

Solution:

From the table, we have $P(x=0)=\dfrac{2}{5}$, $P(x=1)=\dfrac{8}{15}$ and we also know that $\sum P(x)=1$. So

$$\begin{aligned}
\sum P(x) &= 1\\
P(x=0)+P(x=1)+P(x=2) &= 1\\
\frac{2}{5}+\frac{8}{15}+P(x=2) &= 1\\
\frac{14}{15}+P(x=2) &= 1\\
P(x=2) &= 1-\frac{14}{15}\\
P(x=2) &= \frac{1}{15}
\end{aligned}$$

What is a **Discrete Probability Distribution**?

It is a probability distribution for a discrete random variable x with probability $P(x)$ such that

- $\sum P(x) = 1$, and
- $0 \leq P(x) \leq 1$.

Finding **Mean**, **Variance**, and **Standard Deviation**:

- Mean $\Rightarrow \mu = \sum \left[x \cdot P(x)\right]$,
- Variance $\Rightarrow \sigma^2 = \sum \left[x^2 \cdot P(x)\right] - \mu^2$, and
- Standard Deviation $\Rightarrow \sigma = \sqrt{\sigma^2}$.

Example:

Let x be the number of boys in a family with 4 children and $P(x)$ be the probability of the number of boys.

x	$P(x)$
0	.0625
1	.2500
2	.3750
3	.2500
4	.0625

Use the table above to find the mean, variance, and standard deviation of the number of boys in a family with 4 children.

Solution:

We begin by extending our table and compute $\sum x \cdot P(x)$, and $\sum x^2 \cdot P(x)$.

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	.0625	$0 \cdot .0625$	$0^2 \cdot .0625$
1	.2500	$1 \cdot .2500$	$1^2 \cdot .2500$
2	.3750	$2 \cdot .3750$	$2^2 \cdot .3750$
3	.2500	$3 \cdot .2500$	$3^2 \cdot .2500$
4	.0625	$4 \cdot .0625$	$4^2 \cdot .0625$
Total		2	5

Now Using the formula, we get $\mu = 2$, $\sigma^2 = 1$, and $\sigma = 1$.

---

we can use technology to find these values as these calculations have a great tendency to get nasty quickly.

Example:

In a survey of 250 randomly selected registered students in a summer session, 35 students were taking 3 units, 75 students were taking 4 units, 95 students were taking 5 units, and the rest were taking 6 units. Complete the following table

Number of units	Probability of the Number of Units

and then find the mean, variance, and standard deviation of the number of units students are taking in a summer session.

Solution:

We will first compute the probabilities from the information in the survey. Let x be the number of units taken by a student so $P(x = 3) = \frac{35}{250} = 0.14$, $P(x = 4) = \frac{75}{250} = 0.30$, and so on.

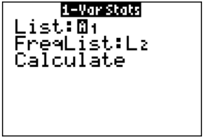
We are now ready to complete the table.

x(number of units)	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
3	0.14	$3 \cdot 0.14$	$3^2 \cdot 0.14$
4	0.30	$4 \cdot 0.30$	$4^2 \cdot 0.30$
5	0.38	$5 \cdot 0.38$	$5^2 \cdot 0.30$
6	0.18	$6 \cdot 0.18$	$6^2 \cdot 0.18$
Total		4.6	22.04

Now using the formulas, we get $\mu = 4.6$, $\sigma^2 = 0.88$, and $\sigma = 0.938$.

Discrete Probability Distributions & TI

- Organize the given information in a table in two columns with heading x and $P(x)$ such that
- Enter all x values in L_1.
- Enter all $P(x)$ values in L_2.
- Execute the following command as they fit your type of calculator to find $\mu = \bar{x}$ and $\sigma = \sigma_x$.



What is **Expected Value**?

Expected Value is a measurement of the mean of a probability distribution.

How do we compute **Expected Value**?

If x is a discrete random variable which has a discrete probability distribution , then the expected value is defined by

$$\begin{aligned} E(x) &= \mu \\ &= \sum x \cdot P(x) \end{aligned}$$

Example:

At a college fundraising event, the math club sold 400 tickets for \$2 each. The winning ticket will receive a brand new calculator valued at \$125. What is the expected value for net earning per ticket for the math club?

Solution:

Since there is only one winning ticket, then $P(Win) = \frac{1}{400}$ and $P(\overline{Win}) = \frac{399}{400}$. The amount of net winning for the winning ticket is $\$125 - \$2 = \$123$ while the rest of the tickets have a net winning of $-\$2$, therefore the expected value is

$$\mu = 123 \cdot \frac{1}{400} - 2 \cdot \frac{399}{400} \approx \$ - 1.69 \text{ per ticket}$$

What is **Binomial Probability Distribution**?

It is a probability distribution for a discrete random variable x with probability $P(x)$ such that

- $\sum P(x) = 1$.
- $0 \leq P(x) \leq 1$.
- It has a fixed number of independent events.
- Each event has only two outcomes, and are referred to as success and failure.
- The probability of success and failure remains the same for all events.

Notation for **Binomial Probability Distributions**:

- Number of independent trials $\Rightarrow n$
- Number of successes $\Rightarrow x$
- Probability of success in one of the trials $\Rightarrow p$
- Probability of failure in one of the trials $\Rightarrow q$ where $p + q = 1$

Formula for **Binomial Probability Distributions**:

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x} \text{ for } x = 0, 1, 2, 3, \cdots, n$$

Example:

Given: Binomial probability distribution with $n = 25$, and $p = .8$. Find $P(x = 18)$.

Solution:

We first compute
$q = 1 - p = 1 - 0.8 = 0.2$,
${}_{25}C_{18} = 480700$, and
$n - x = 25 - 18 = 7$.
Now we can use the binomial probability distribution formula to find $P(x = 18)$.

$$\begin{aligned} P(x = 18) &= {}_{25}C_{18} \cdot (.8)^{18} \cdot (.2)^{7} \\ &\approx 0.1108 \end{aligned}$$

Binomial Probability Distributions Keywords

Keyword	Translation
exactly	$P(x = a)$
at most	$P(x \leq a)$
at least	$P(x \geq a)$
fewer than	$P(x < a)$
more than	$P(x > a)$
between a and b, inclusive	$P(a \leq x \leq b)$
from a to b	$P(a \leq x \leq b)$

Example:

You are about to take a multiple-choice test with 20 questions. Each question has 5 possible choices but only one choice is correct. If you were to guess randomly on all questions,

1. what is the probability of guessing exactly 3 correct answers?
2. what is the probability of guessing fewer than 4 correct answers?
3. what is the probability of guessing at least 3 correct answers?

Solution:

We first obtain from the problem that $n = 20$, $p = \frac{1}{5} = 0.2$, and $q = \frac{4}{5} = 0.8$. Now

Solution Continued:

1. what is the probability of guessing exactly 3 correct answers? $\Rightarrow P(x = 3)$, so

$$\begin{aligned} P(x = 3) &= {}_{20}C_3 \cdot (.2)^3 \cdot (.8)^{17} \\ &\approx 0.205 \end{aligned}$$

2. what is the probability of guessing fewer than 4 correct answers? $\Rightarrow P(x < 4) = P(x \leq 3)$, so

$$\begin{aligned} P(x \leq 3) &= P(x = 0) + \cdots + P(x = 3) \\ &\approx 0.411 \end{aligned}$$

3. what is the probability of guessing at least 3 correct answers? $\Rightarrow P(x \geq 3)$, so

$$\begin{aligned} P(x \geq 3) &= 1 - P(x \leq 2) \\ &= 1 - [P(x = 0) + P(x = 1) + P(x = 2)] \\ &\approx 0.794 \end{aligned}$$

Example:

An airline claims that 98% of passengers with tickets actually show up for the flight. The airline overbooked a flight by selling 145 tickets for the aircraft that has 138 seats. What is the probability that the number of passengers with tickets who show up is between 135 and 138, inclusive?

Solution:

We first obtain from the problem that $n = 145$, $p = 0.98$, and $q = 0.02$. Since we want the probability that the number of passengers with tickets show up is between 135 and 138, inclusive, this translates to $P(135 \leq x \leq 138)$.

$$\begin{aligned} P(135 \leq x \leq 138) &= P(x = 135) + \cdots + P(x = 138) \\ &\approx 0.709 \end{aligned}$$

Binomial Probability Distributions & TI

Binomial Probability	TI Command
$P(x = a)$	binompdf(n, p, a)
$P(x \leq a)$	binomcdf(n, p, a)
$P(x \geq a)$	1−binomcdf$(n, p, a - 1)$
$P(x < a)$	binomcdf$(n, p, a - 1)$
$P(x > a)$	1−binomcdf(n, p, a)
$P(a \leq x \leq b)$	binomcdf(n, p, b)−binomcdf$(n, p, a - 1)$

What is the probability of that?