Basic Probability

Terminologies & Properties

What is a **Probability**?

Probability is a branch of mathematics that deals with calculating the likelihood of a given event to happen or not, which is expressed as a number from 0 to 1.

What is an **Event**?

An **Event** is any collection of outcomes of a procedure.

What is a **Simple Event**?

An **Event** that cannot be further broken down into simpler components.

What is a **Sample Space**?

Sample Space is a collection of all possible simple events of a procedure.

Example:

Find the sample space for the following procedures.

1. Single birth
2. Flip a coin twice
3. Flip a coin followed by rolling a four–sided die

Solution:

1. Single birth $\Longrightarrow$ let's use B to denote a boy and G to denote a girl, then the sample space is $\{B, G\}$.

2. Flip a coin twice $\Longrightarrow$ let's use H to denote heads outcome and T to denote tails outcome, then the sample space is $\{HH, HT, TH, TT\}$.

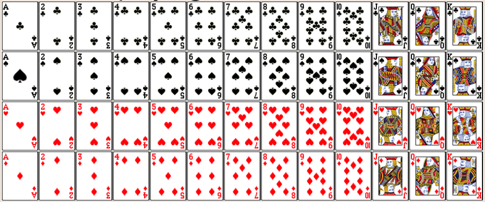
3. Flip a coin followed by rolling a four–sided die $\Longrightarrow$ let's use H to denote heads outcome and T to denote tails outcome along with numbers $1, 2, 3, 4$ for the outcomes of the four-sided die then the sample space is $\{H1, H2, H3, H4, T1, T2, T3, T4\}$.

How do we find the **Probability** of an **Event**?

$$\textbf{Probability}(\text{Desired Event}) = \frac{\text{The number of desired outcomes}}{\text{The number of all possible outcomes}}$$

Example:

Consider a full-deck of playing cards shown below.



What is the probability of randomly drawing an ace?

What is the probability of randomly drawing a face card?

Solution:

$$\begin{aligned}\text{Probability(Draw an ace)} &= \frac{\text{Number of aces}}{\text{Total number of cards}} \\ &= \frac{4}{52} = \frac{1}{13} \\ &\approx 0.077\end{aligned}$$

$$\begin{aligned}\text{Probability(Draw a face card)} &= \frac{\text{Number of face cards}}{\text{Total number of cards}} \\ &= \frac{12}{52} = \frac{3}{13} \\ &\approx 0.231\end{aligned}$$

What are the properties of **Probability**?

Let E be all possible events, A be the desired event with $P(E)$ and $P(A)$ be the corresponding probabilities,

- $0 \leq P(A) \leq 1$
- $\sum P(E) = 1$
- $\bar{A}$ is the complement of the event A, which means not A.
- $P(\bar{A}) + P(A) = 1$, or $P(\bar{A}) = 1 - P(A)$

Example:

Which of the following values cannot be probabilities?

$$\frac{7}{5}, -0.75, 125\%$$

Solution:

None of these values can be used to express the probabilities since they do not satisfy $0 \leq P(A) \leq 1$.

Example:

Find $P(\bar{A})$ if $P(A) = .05$.

Solution:

Since $P(\bar{A}) = 1 - P(A)$, so $P(\bar{A}) = 1 - 0.05$ then $P(\bar{A}) = 0.95$.

What is a **Sure Event**?

Event A is considered a **Sure Event** if $P(A) = 1$.

Example:

Suppose you roll a normal die. What is the probability that you will get a number less than 7?

Solution:

The probability that you will get a number less than 7 is 1 since any outcome is a number less than 7. The event is a sure event .

What is an **Impossible Event**?

Event A is considered an **Impossible Event** if $P(A) = 0$.

Example:

What is the probability that someone is born on February 30th?

Solution:

The probability that someone is born on February 30th is 0 since there is no such date on the calendar. The event is impossible .

What is a **Rare Event**?

Event A is considered a **Rare Event** if $0 < P(A) \leq .05$.

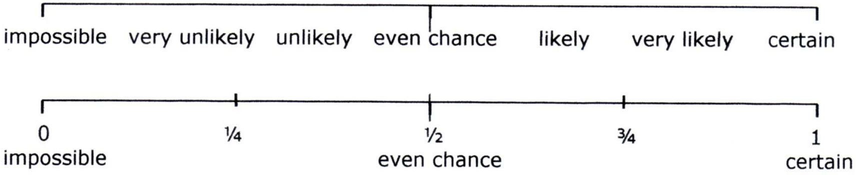
Example:

What is the probability that randomly selected person has a birthday today?

Solution:

The probability that anyone randomly selected has a birthday today is $\dfrac{1}{365} \approx 0.003$ since that is less than .05, it is a rare event.

Basic Probability Scale



Example:

Suppose a red fair die and a white fair die is rolled. The display below shows all possible outcomes.

		White Die					
		1	2	3	4	5	6
Red Die	1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
	2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
	3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
	4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
	5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
	6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

1. List all possible sums.
2. What is the probability that the sum of the outcomes is 1?
3. What is the probability that the sum of the outcomes is between 2 and 12, inclusive?

Solution:

1. List all possible sums $\Rightarrow \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
2. $P(Sum = 1) = 0$ since there is no outcome with the sum of 1.
3. $P(2 \leq Sum \leq 12) = 1$ since the sum of any outcomes is between 2 and 12, inclusive.

Example:

Use the last example to complete the following table

Sum	2	3	4	5	6	7	8	9	10	11	12
P(Sum)											

then verify that $\sum P(Sum) = 1$.

Solution:

There are 36 outcomes altogether,

$P(Sum = 2) = P((1,1)) = \frac{1}{36}$, $P(Sum = 12) = P((6,6)) = \frac{1}{36}$

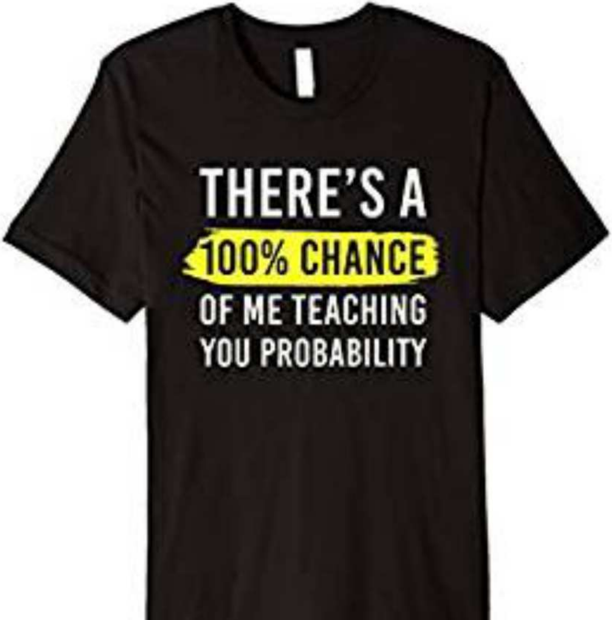
$P(Sum = 3) = P((1,2),(2,1)) = \frac{2}{36}$, $P(Sum = 11) = P((6,5),(5,6)) = \frac{2}{36}$

$P(Sum = 4) = P((1,3),(2,2),(3,1)) = \frac{3}{36}$

We continue this to get the rest of the probabilities.

Sum	2	3	4	5	6	7	8	9	10	11	12
P(Sum)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

It is easier to verify that $\sum P(Sum) = 1$ if these probabilities are not reduce.


THERE'S A
100% CHANCE
OF ME TEACHING
YOU PROBABILITY