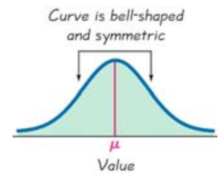
Chapter 6
Normal Probability Distributions

Overview

- Continuous random variable
- Normal distribution



Definitions

- **Uniform Distribution** is a probability distribution in which the continuous random variable values are spread evenly over the range of possibilities; the graph results in a rectangular shape.

Definitions

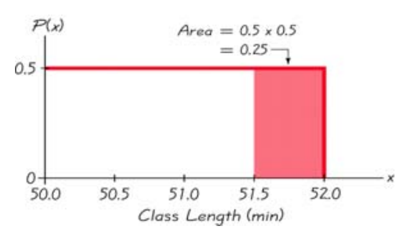
- **Density Curve** (or probability density function is the graph of a continuous probability distribution.

1. The total area under the curve must equal 1.

2. Every point on the curve must have a vertical height that is 0 or greater.

Because the total area under the density curve is equal to 1, there is a correspondence between area and probability.

Using Area to Find Probability



Is the total area equal to 1?

Heights of Adult Men and Women

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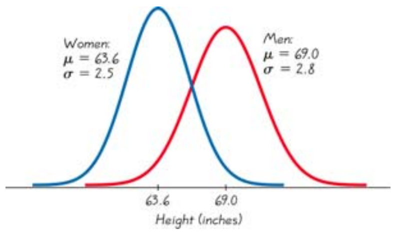

Why is the red curve on the right?

Why does the blue curve have higher height?

Definition

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Standard Normal Distribution:

a normal probability distribution that has a mean of 0 and a standard deviation of 1, and the total area under its density curve is equal to 1.

Definition

Slide 9

Standard Normal Distribution:

a normal probability distribution that has a mean of 0 and a standard deviation of 1.

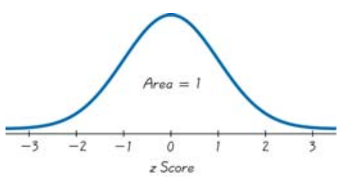

Figure 5-5

Notation

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P($a < z < b$)

denotes the probability that the z score is between a and b

P($z > a$)

denotes the probability that the z score is greater than a

P($z < a$)

denotes the probability that the z score is less than a

Table A-2

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- **Inside front cover of text book**
- **Formulas and Tables card**
- **Appendix**

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TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051 *	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505 *	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

To find:

z Score

_the <u>distance</u> along horizontal scale of the standard normal distribution; refer to the <u>leftmost column and top row</u> of Table A-2.

Area

_the <u>region</u> under the curve; refer to the values in the <u>body</u> of Table A-2.

Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than 1.58 degrees.

$P(z < 1.58) =$

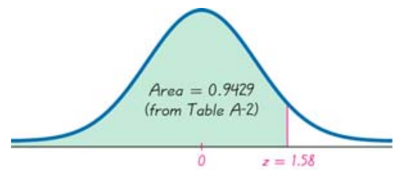


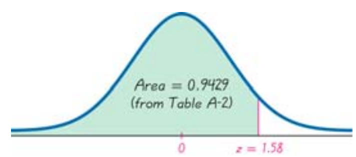
Figure 5-6

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890

Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than 1.58 degrees.

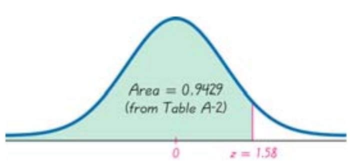
$P(z < 1.58) = 0.9429$



The probability that the chosen thermometer will measure freezing water less than 1.58 degrees is 0.9429.

Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than 1.58 degrees.

$P(z < 1.58) = 0.9429$



94.29% of the thermometers have readings less than 1.58 degrees.

Using TI:
Standard Normal Distribution

$$P(z < a)$$

1) 2nd VARS(DISTR)
2) Arrow down to normalcdf(
3) enter
4) VSNN , a , 0 , 1) enter

Mean — Standard Deviation

VSNN→ Very Small Negative Number

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Using TI: Standard Normal Distribution

Example: **Find P(z<1.58)**

1) Select 2nd, VARS, arrow down to get to **normalcdf(** enter to select

```
DISTR DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:tpdf(
5:tcdf(
6:X²pdf(
7↓X²cdf(
```

2) key in

-10000, 1.58,0,1)

```
normalcdf(-1000,
1.58,0,1)
```


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Using TI: Standard Normal Distribution

Example: **Find P(z<1.58)**

3) Enter to execute this operation and get the final answer.

```
normalcdf(-1000,
1.58,0,1)
          .942946563
```

This result was obtained earlier by directly using the Standard Normal Distribution table.

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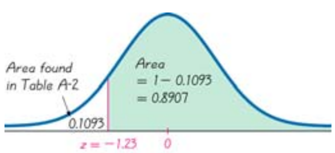
Example:

If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above –1.23 degrees.

P (z > –1.23) = 0.8907



The probability that the chosen thermometer with a reading above –1.23 degrees is 0.8907.

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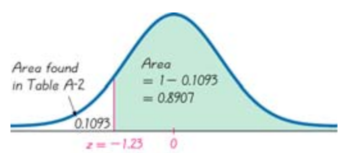
Example:

If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above –1.23 degrees.

P (z > –1.23) = 0.8907



89.07% of the thermometers have readings above –1.23 degrees.

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Using TI: Standard Normal Distribution

$$P(z > a)$$

1) **2nd VARS(DISTR)**

2) **Arrow down to normalcdf(**

3) **enter**

4) **a , VLPN , 0 , 1) enter**

Mean | Standard Deviation

VLPN→ Very Large Positive Number

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Using TI: Standard Normal Distribution

Example: **Find P(z> – 1.23)**

1) Select 2nd, VARS, arrow down to get to **normalcdf(** enter to select

```
DISTR DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:tpdf(
5:tcdf(
6:X²pdf(
7↓X²cdf(
```

2) key in

– 1.23, 1000 , 0 , 1)

```
normalcdf(-1.23,
1000,0,1)
```

Using TI: Standard Normal Distribution

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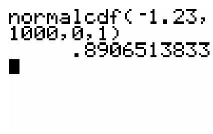
Example: Find P(z> – 1.23)

3) Enter to execute this operation and get the final answer.



This result was obtained earlier by directly using the Standard Normal Distribution table.

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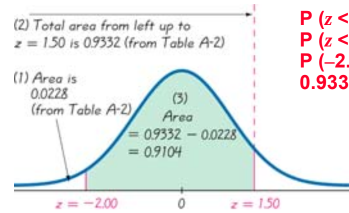
Example: **A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.**



P (z < –2.00) = 0.0228
P (z < 1.50) = 0.9332
P (–2.00 < z < 1.50) = 0.9332 – 0.0228 = 0.9104

The probability that the chosen thermometer has a reading between - 2.00 and 1.50 degrees is 0.9104.

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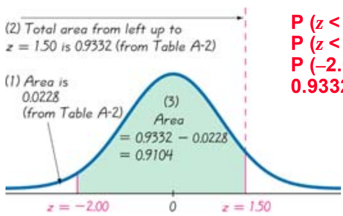
Example: **A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.**



P (z < –2.00) = 0.0228
P (z < 1.50) = 0.9332
P (–2.00 < z < 1.50) = 0.9332 – 0.0228 = 0.9104

If many thermometers are selected and tested at the freezing point of water, then 91.04% of them will read between –2.00 and 1.50 degrees.

Using TI: Standard Normal Distribution

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$P(a < z < b)$

1) 2nd VARS(DISTR)

2) Arrow down to normalcdf(

3) enter

4) a , b , 0 , 1) enter

Mean

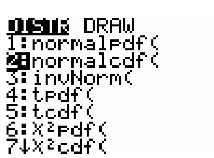
Standard Deviation

Using TI: Standard Normal Distribution

Slide 29

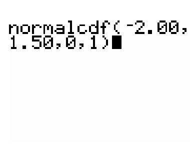
Example: Find P(– 2.00 < z < 1.50)

1) Select 2nd, VARS, arrow down to get to **normalcdf(** enter to select



2) key in

– 2.00, 1.50 , 0 , 1)



Using TI: Standard Normal Distribution

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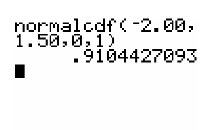
Example: Find P(– 2.00 < z < 1.50)

3) Enter to execute this operation and get the final answer.



This result was obtained earlier by directly using the Standard Normal Distribution table.

Finding a *z* - score when given a probability Using Table A-2

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1. Draw a bell-shaped curve, draw the centerline, and identify the region under the curve that corresponds to the given probability. If that region is not a cumulative region from the left, work instead with a known region that is a cumulative region from the left.
2. Using the cumulative area from the left, locate the closest probability in the ***body*** of Table A-2 and identify the corresponding *z* score.

Finding *z* Scores when Given Probabilities

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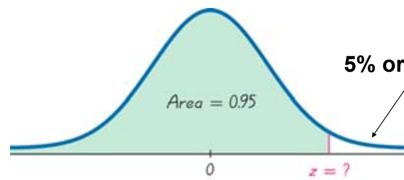

5% or 0.05

(*z* score will be positive)

Figure 5-10
Finding the 95th Percentile

Finding *z* Scores when Given Probabilities

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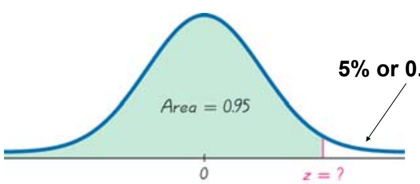

5% or 0.05

1.645

(*z* score will be positive)

Figure 5-10
Finding the 95th Percentile

Using TI: Standard Normal Distribution

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$P(z < a) = LMA$

1) 2nd VARS(DISTR)
2) Arrow down to invNorm(
3) enter
4) LMA, 0 , 1) enter

Mean — Standard Deviation

LMA → Left Most Area

Using TI: Standard Normal Distribution

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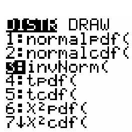
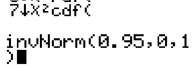
Example: **Find the z - score if P(z < a)=0.95**

1) Select 2nd, VARS, arrow down to get to **invNorm(** enter to select



2) key in

0.95 , 0 , 1)



Using TI: Standard Normal Distribution

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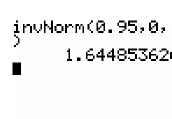
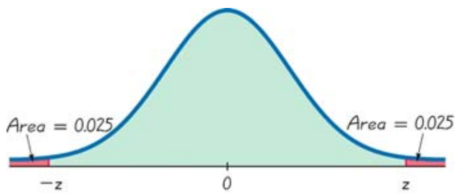
Example: **Find the z - score if P(z < a)=0.95**

3) Enter to execute this operation and get the final answer.



This result was obtained earlier by directly using the Standard Normal Distribution table.

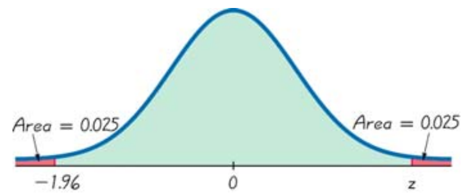
Finding z Scores when Given Probabilities



(One z score will be negative and the other positive)

Figure 5-11
Finding the Bottom 2.5% and Upper 2.5%

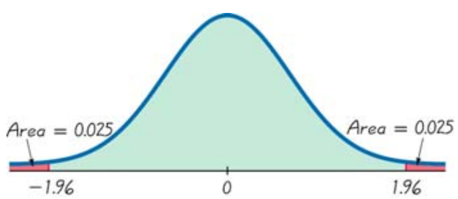
Finding z Scores when Given Probabilities



(One z score will be negative and the other positive)

Figure 5-11
Finding the Bottom 2.5% and Upper 2.5%

Finding z Scores when Given Probabilities



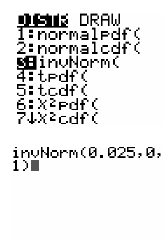
(One z score will be negative and the other positive)

Figure 5-11
Finding the Bottom 2.5% and Upper 2.5%

Using TI: Standard Normal Distribution

Example: Find the z - score if P(z < a)=0.025

1) Select 2nd, VARS, arrow down to get to **invNorm(** enter to select



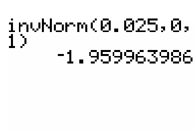
2) key in

0.025 , 0 , 1)

Using TI: Standard Normal Distribution

Example: Find the z - score if P(z < a)=0.025

3) Enter to execute this operation and get the final answer.



This result was obtained earlier by directly using the Standard Normal Distribution table.

Nonstandard Normal Distributions

If $\mu \neq 0$ or $\sigma \neq 1$ (or both), we will convert values to standard scores using Formula 5-2, then procedures for working with all normal distributions are the same as those for the standard normal distribution.

Formula 5-2 $$z = \frac{x - \mu}{\sigma}$$

Converting to Standard Normal Distribution

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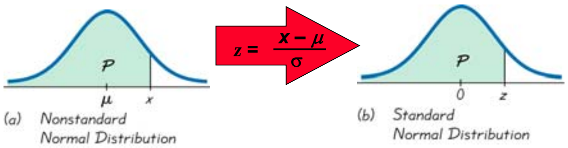

Figure 5-12

Probability of Sitting Heights Less Than 38.8 Inches

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- The sitting height (from seat to top of head) of drivers must be considered in the design of a new car model. Men have sitting heights that are normally distributed with a mean of 36.0 in. and a standard deviation of 1.4 in. (based on anthropometric survey data from Gordon, Clauser, et al.). Engineers have provided plans that can accommodate men with sitting heights up to 38.8 in., but taller men cannot fit. If a man is randomly selected, find the probability that he has a sitting height less than 38.8 in. Based on that result, is the current engineering design feasible?

Probability of Sitting Heights Less Than 38.8 Inches

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$\mu =$ 36.0

$\sigma =$ 1.4

$$z = \frac{38.8 - 36.0}{1.4} = 2.00$$

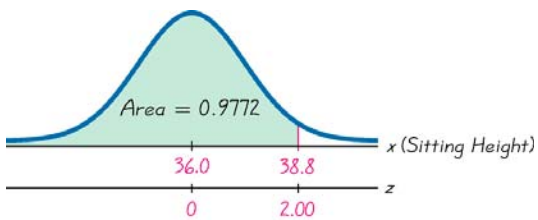

Figure 5-13

Probability of Sitting Heights Less Than 38.8 Inches

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$\mu =$ 36.0

$\sigma =$ 1.4

P (x < 38.8 in.) = P(z < 2)
= 0.9772

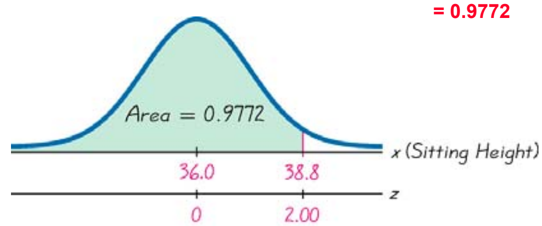

Figure 5-13

Using TI: Non-Standard Normal Distribution

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$$P(x < a)$$

1) 2nd VARS(DISTR)

2) Arrow down to normalcdf(

3) enter

4) VSNN , a , μ , σ) enter

Mean | Standard Deviation

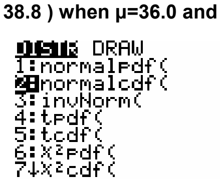
VSNN→ Very Small Negative Number

Using TI: Standard Normal Distribution

Slide 48

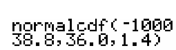
Example: Find P(x < 38.8) when μ=36.0 and σ=1.4.

1) Select 2nd, VARS, arrow down to get to **normalcdf(** enter to select



2) key in

–1000 , 38.8 , 36.0,1.4)



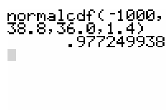
Mean | Standard Deviation

Using TI: Non-Standard Normal Distribution

Slide 49

Example: Find P(x < 38.8) when μ=36.0 and σ=1.4.

3) Enter to execute this operation and get the final answer.



This result was obtained earlier by converting to the Standard Normal Distribution and then using the table.

Probability of Weight between 140 pounds and 211 pounds

Slide 50

In the Chapter Problem, we noted that the Air Force had been using the ACES-II ejection seats designed for men weighing between 140 lb and 211 lb. Given that women's weights are normally distributed with a mean of 143 lb and a standard deviation of 29 lb (based on data from the National Health survey), what percentage of women have weights that are within those limits?

Probability of Weight between 140 pounds and 211 pounds

Slide 51

$\mu = 143$
$\sigma = 29$

$$z = \frac{211 - 143}{29} = 2.34$$

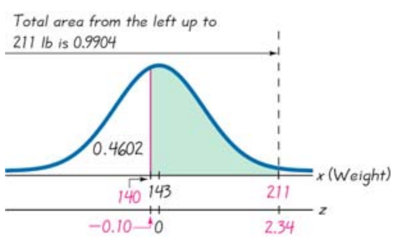


Figure 5-14

Probability of Weight between 140 pounds and 211 pounds

Slide 52

$\mu = 143$
$\sigma = 29$

$$z = \frac{140 - 143}{29} = -0.10$$

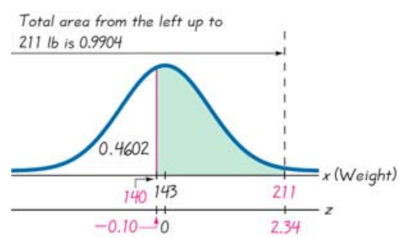


Figure 5-14

Probability of Weight between 140 pounds and 211 pounds

Slide 53

$\mu = 143$
$\sigma = 29$

$P(-0.10 < z < 2.34) =$

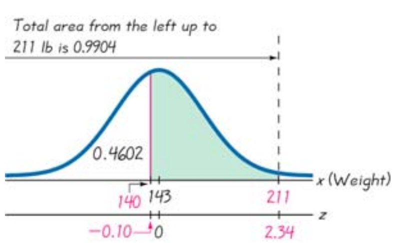


Figure 5-14

Probability of Weight between 140 pounds and 211 pounds

Slide 54

$\mu = 143$
$\sigma = 29$

$0.9904 - 0.4602 = 0.5302$

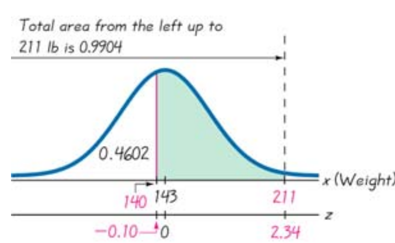


Figure 5-14

Probability of Weight between 140 pounds and 211 pounds

Slide 55

$\mu =$ 143
$\sigma =$ 29

There is a 0.5302 probability of randomly selecting a woman with a weight between 140 and 211 lbs.

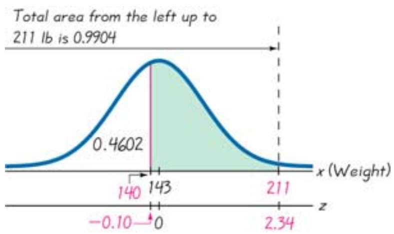


Figure 5-14

Probability of Weight between 140 pounds and 211 pounds

Slide 56

$\mu =$ 143
$\sigma =$ 29

OR - 53.02% of women have weights between 140 lb and 211 lb.

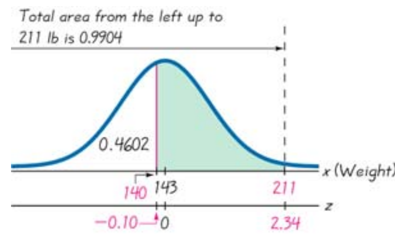


Figure 5-14

Using TI: Non-Standard Normal Distribution

Slide 57

$P(a < x < b)$

1) 2nd VARS(DISTR)

2) Arrow down to normalcdf(

3) enter

4) a , b , μ , σ) enter

Mean

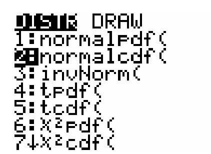
Standard Deviation

Using TI: Non-Standard Normal Distribution

Slide 58

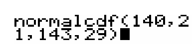
Example: Find P(140 < x < 211) when μ=143 & σ=29.

1) Select 2nd, VARS, arrow down to get to **normalcdf(** enter to select



2) key in

140 , 211 , 143 , 29)

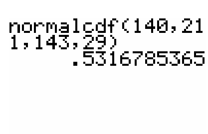


Using TI: Non-Standard Normal Distribution

Slide 59

Example: Find P(140 < x < 211) when μ=143 & σ=29.

3) Enter to execute this operation and get the final answer.



This result was obtained earlier by converting to the Standard Normal Distribution and then using the table.

Why is this answer slightly different from the earlier method?

Finding a *z* - score when given a probability Using Table A-2

Slide 60

1. **Draw a bell-shaped curve, draw the centerline, and identify the region under the curve that corresponds to the given probability. If that region is not a cumulative region from the left, work instead with a known region that is a cumulative region from the left.**
2. **Using the cumulative area from the left, locate the closest probability in the *body* of Table A-2 and identify the corresponding *z* score.**

Cautions to keep in mind

Slide 61

1. Don't confuse z scores and areas. z scores are distances along the horizontal scale, but areas are regions under the normal curve. Table A-2 lists z scores in the left column and across the top row, but areas are found in the body of the table.
2. Choose the correct (right/left) side of the graph.
3. A z score must be negative whenever it is located to the *left* half of the normal distribution.
4. Areas (or probabilities) are positive or zero values, but they are never negative.

Procedure for Finding Values Using Table A-2 and Formula 5-2

Slide 62

1. Sketch a normal distribution curve, enter the given probability or percentage in the appropriate region of the graph, and identify the x value(s) being sought.
2. Use Table A-2 to find the z score corresponding to the cumulative left area bounded by x. Refer to the BODY of Table A-2 to find the closest area, then identify the corresponding z score.
3. Using Formula 5-2, enter the values for μ, σ, and the z score found in step 2, then solve for x.

$$x = \mu + (z \cdot \sigma)$$ (Another form of Formula 5-2)

(If z is located to the left of the mean, be sure that it is a negative number.)

4. Refer to the sketch of the curve to verify that the solution makes sense in the context of the graph and the context of the problem.

Find P_{98} for Hip Breadths of Men

Slide 63

$z = 2.05$

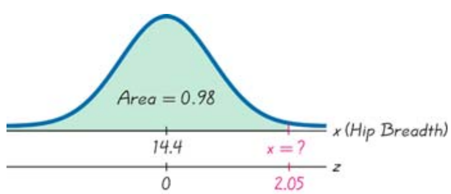


Figure 5-15

Find P_{98} for Hip Breadths of Men

Slide 64

$x = \mu + (z \bullet \sigma)$
$x = 14.4 + (2.05 \bullet 1.0)$
$x = 16.45$

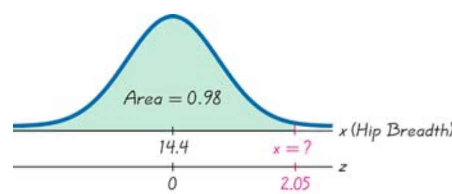


Figure 5-15

Find P_{98} for Hip Breadths of Men

Slide 65

The hip breadth of 16.5 in. separates the lowest 98% from the highest 2%

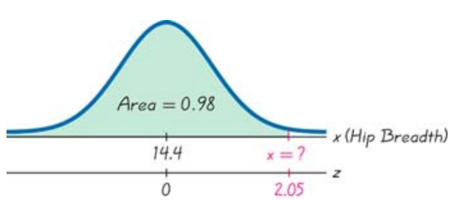


Figure 5-15

Find P_{98} for Hip Breadths of Men

Slide 66

Seats designed for a hip breadth up to 16.5 in. will fit 98% of men.

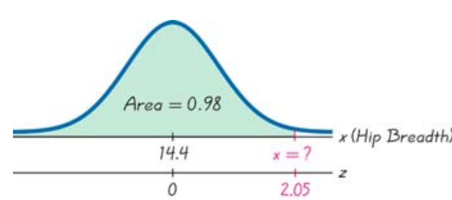


Figure 5-15

Using TI: Non-Standard Normal Distribution

Slide 67

$P(x < a) = LMA$

1) 2nd VARS(DISTR)
2) Arrow down to invNorm(
3) enter
4) LMA, μ , σ) enter

Mean — Standard Deviation

LMA → Left Most Area

Using TI: Non-Standard Normal Distribution

Slide 68

Example: Find the x - score if P(x < a)=0.98 when μ=14.4 and σ=1.0.

1) Select 2nd, VARS, arrow down to get to **invNorm(** enter to select

```
DISTR DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:tpdf(
5:tcdf(
6:X²pdf(
7↓X²cdf(
```

2) key in

0.98 , 14.4 , 1.1)

```
invNorm(.98,14.4
,1.0)
```

Mean — Standard deviation

Using TI: Standard Normal Distribution

Slide 69

Example: Find the x - score if P(x < a)=0.98 when μ=14.4 and σ=1.0.

3) Enter to execute this operation and get the final answer.

```
invNorm(.98,14.4
,1.0)
        16.45374891
```

This result was obtained earlier by directly using the Standard Normal Distribution table.

Finding P_{05} for Grips of Women

Slide 70

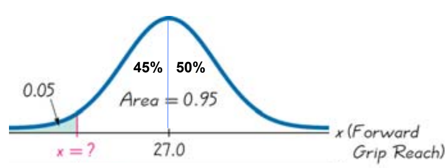


Figure 5-16

Finding P_{05} for Grips of Women

Slide 71

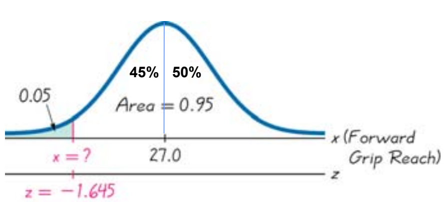


Figure 5-16

Finding P_{05} for Grips of Women

Slide 72

$x = 27.0 + (-1.645 \cdot 1.3) = 24.8615$

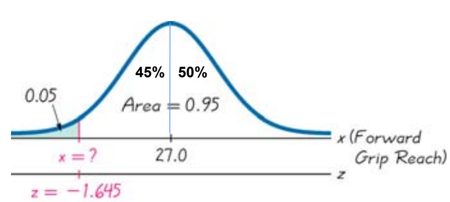


Figure 5-16

Finding P_{05} for Grips of Women

The forward grip of 24.9 in. (rounded) separates the top 95% from the others.

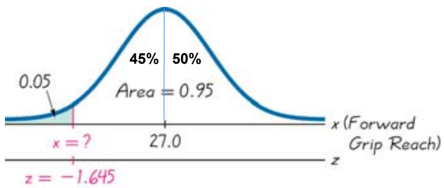


Figure 5-16

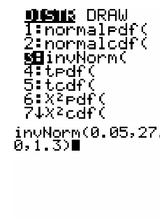
Using TI: Non-Standard Normal Distribution

Example: **Find the x - score if P(x < a)=0.05 when μ=27.0 and σ=1.3.**

1) Select 2nd, VARS, arrow down to get to **invNorm(** enter to select

2) key in

0.05 , 27.0 , 1.3)



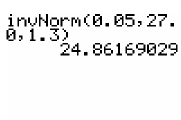
Mean

Standard deviation

Using TI: Non-Standard Normal Distribution

Example: **Find the x - score if P(x < a)=0.05 when μ=27.0 and σ=1.3.**

3) Enter to execute this operation and get the final answer.



This result was obtained earlier by directly using the Standard Normal Distribution table.

REMEMBER!

Make the *z* score negative if the value is located to the left (below) the mean. Otherwise, the *z* score will be positive.

Definition

Sampling Distribution of the mean is the probability distribution of sample means, with all samples having the same sample size *n*.

Definition

Sampling Variability:

The value of a statistic, such as the sample mean $\bar{x}$, depends on the particular values included in the sample.

Consider the population of 2, 4, and 6.
Select sample of size 2 with replacement.

Sample	Sample Mean
2 , 2	2
2 , 4	3
2 , 6	4
4 , 2	3
4 , 4	4
4 , 6	5
6 , 2	4
6 , 4	5
6 , 6	6

Sample Mean	Probability
2	1/9
3	2/9
4	3/9
5	2/9
6	1/9

Consider the population of 2, 4, and 6.
Select sample of size 2 with replacement.

Now

use your calculator to compute the mean and standard deviation of the sample means.

a) Enter sample means into L1

b) Enter corresponding probabilities into L2.

c) Stat, Calc, 1-var stat, L1, L2, enter

Now

a) Enter element of the population into L3.

b) Stat, Calc, 1-var stat, L1, L2, enter

Consider the population of 2, 4, and 6.
Select sample of size 2 with replacement.

Did you notice that the mean of the sample means is equal to the mean of the population?

$$\mu_{\bar{X}} = \mu$$

Now divide the population standard deviation by the square root of the each sample size, in this case 2.

Is this answer familiar to you?

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Interpretation of Sampling Distributions

We can see that when using a sample statistic to estimate a population parameter, some statistics are good in the sense that they target the population parameter and are therefore likely to yield good results. Such statistics are called *unbiased estimators*.

Statistics that target population parameters: mean, variance, proportion

Statistics that do not target population parameters: median, range, standard deviation

Central Limit Theorem

Given:

1. The random variable x has a distribution (which may or may not be normal) with mean μ and standard deviation σ.

2. Samples all of the same size n are randomly selected from the population of x values.

Central Limit Theorem

Conclusions:

1. The distribution of the sample means will, as the sample size increases, approach a normal distribution.

2. The mean of the sample means will be the population mean μ.

3. The standard deviation of the sample means will approach $\frac{\sigma}{\sqrt{n}}$.

Practical Rules Commonly Used:

Slide 85

1. For samples of size n larger than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation gets better as the sample size n becomes larger.

2. If the original population is itself normally distributed, then the sample means will be normally distributed for any sample size n (not just the values of n larger than 30).

Notation

Slide 86

the mean of the sample means

$$\mu_{\bar{x}} = \mu$$

the standard deviation of sample mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(often called standard error of the mean)

Using TI

Slide 87

When using normalcdf with sample size $n = 1$, enter the following four enteries in the order: normalcdf(LV, RV, μ, σ).

Example: $P(x > 24)$ when $\mu = 26$ & $\sigma = 1.5$

$$P(x > 24) = \text{normalcdf}(24, 1000,\ 26, 1.5)$$

Answer:

Using TI

Slide 88

When using normalcdf with sample size $n > 1$, enter the following four enteries in the order: normalcdf(LV, RV, μ, $\frac{\sigma}{\sqrt{n}}$).

Example:

$P(\bar{x} > 24)$ when $n = 10$, $\mu = 26$ & $\sigma = 1.5$

$$P(\bar{x} > 24) = \text{normalcdf}(24, 1000,\ 26, \frac{1.5}{\sqrt{10}})$$

Answer:

Slide 89

Example: Given the population of men has normally distributed weights with a mean of 172 lb and a standard deviation of 29 lb,

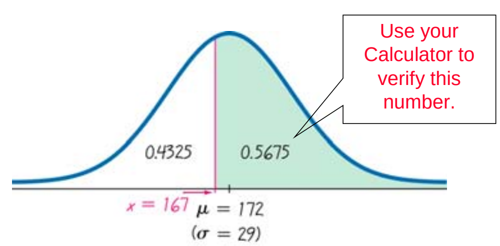
a) if one man is randomly selected, find the probability that his weight is greater than 167 lb.

b) if 12 different men are randomly selected, find the probability that their mean weight is greater than 167 lb.

Slide 90

Example: Given the population of men has normally distributed weights with a mean of 172 lb and a standard deviation of 29 lb,

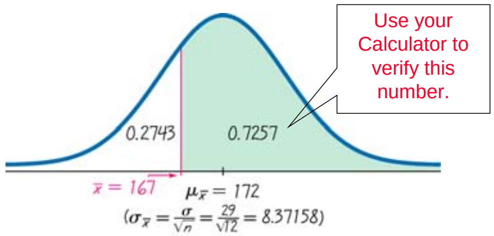
a) if **one man** is randomly selected, the probability that his weight is greater than 167 lb. is 0.5675.



Slide 91

Example: Given the population of men has normally distributed weights with a mean of 172 lb and a standard deviation of 29 lb,

b) if 12 different men are randomly selected, find the probability that their mean weight is greater than 167 lb.



Slide 92

Example: Given the population of men has normally distributed weights with a mean of 172 lb and a standard deviation of 29 lb,

a) if one man is randomly selected, find the probability that his weight is greater than 167 lb.

$P(x > 167) = 0.5675$

b) if 12 different men are randomly selected, their mean weight is greater than 167 lb.

$P(\bar{x} > 167) = 0.7257$

It is much easier for an individual to deviate from the mean than it is for a group of 12 to deviate from the mean.

Slide 93

Sampling Without Replacement

If $n > 0.05\,N$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

finite population correction factor

Slide 94

Approximate a Binomial Distribution with a Normal Distribution if:

$np \geq 5$

$nq \geq 5$

then $\mu = np$ and $\sigma = \sqrt{npq}$

and the random variable has

a  distribution.

(normal)

Slide 95

Procedure for Using a Normal Distribution to Approximate a Binomial Distribution

1. Establish that the normal distribution is a suitable approximation to the binomial distribution by verifying $np \geq 5$ and $nq \geq 5$.
2. Find the values of the parameters μ and σ by calculating $\mu = np$ and $\sigma = \sqrt{npq}$.
3. Identify the discrete value of *x* (the number of successes). Change the discrete value *x* by replacing it with the interval from *x* – 0.5 to *x* + 0.5. Draw a normal curve and enter the values of *μ* , σ, and either *x* – 0.5 or *x* + 0.5, as appropriate.

continued

Slide 96

Procedure for Using a Normal Distribution to Approximate a Binomial Distribution

continued

4. Change *x* by replacing it with *x* – 0.5 or *x* + 0.5, as appropriate.
5. Find the area corresponding to the desired probability.

Definition

When we use the normal distribution (which is continuous) as an approximation to the binomial distribution (which is discrete), a continuity correction is made to a discrete whole number *x* in the binomial distribution by representing the single value *x* by the interval from *x* – 0.5 to *x* + 0.5.

Procedure for Continuity Corrections

1. When using the normal distribution as an approximation to the binomial distribution, *always* use the continuity correction.
2. In using the continuity correction, first identify the discrete whole number *x* that is relevant to the binomial probability problem.
3. Draw a normal distribution centered about μ, then draw a vertical strip area centered over *x* . Mark the left side of the strip with the number *x* – 0.5, and mark the right side with *x* + 0.5. For *x* =120, draw a strip from 119.5 to 120.5. Consider the area of the strip to represent the probability of discrete number *x*.

continued

Procedure for Continuity Corrections

continued

4. Now determine whether the value of *x* itself should be included in the probability you want. Next, determine whether you want the probability of at least *x*, at most *x*, more than *x*, fewer than *x*, or exactly *x*. Shade the area to the right or left of the strip, as appropriate; also shade the interior of the strip itself <u>if and only if</u> *x itself* is to be included. The total shaded region corresponds to probability being sought.

Finding the Probability of "…………." 120 Men Among 200 Accepted Applicants

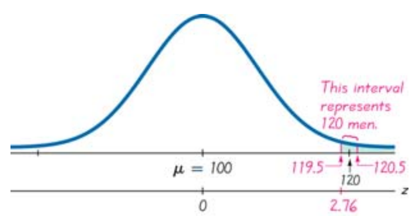
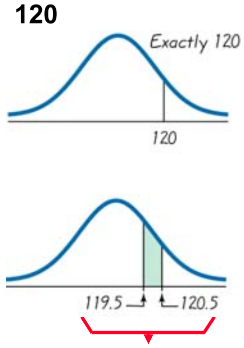


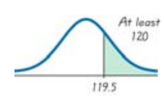
Figure 5-24

***x* = <u>exactly</u> 120**

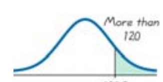


Interval represents discrete number 120

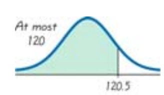
***x* = <u>at least</u> 120**
= 120, 121, 122, . . .



***x* = <u>more than</u> 120**
= 121, 122, 123, . . .



***x* = <u>at most</u> 120**
= 0, 1, . . . 118, 119, 120



***x* = <u>fewer than</u> 120**
= 0, 1, . . . 118, 119

Figure 5-25