Section 12

Motion – Rational Equations

These types of problems are the same concept as the Motion Problems in Section 7, but they involve extra elements and different components.

12.1 Explaining Motion – Rational Equations

In this section, there may be two objects in motion but most often there is just one, traveling at different rates and different distances. The times may be the same, or the time may be given as a total.

Another component is that the rates of the objects are sometimes affected by outside influences such as the speed of water current or the speed of the wind.

12.2 How To Express The Rates

Before you can begin to solve the problem, you need to take note of the rates of any or each object in motion. You name the expressions for the rates in one of two ways. One way is Direct Translation with which you are already familiar.

When Direct Translation is not given, there will be information given about the water current or the wind. This information will be used to determine expressions for the rates.

When water current is involved, the word “current” may be stated in the problem, or words such as “downstream” or “upstream”.

The current affects the rate of an object because when an object is traveling downstream, it means it is going with the current. In that case, the object is being pushed along by the water and would be traveling at a faster speed. However, when an object is traveling upstream, it means it is fighting *against* the water and would be traveling at a slower speed.

The chart below will give you the information you need to name expressions for the rate when current is involved. The variable x represents the rate of an object in still water without the influence of any current. The variable c represents the speed of the current.

Path Of Object	What It Means	What It Does	If x Is Rate In Still Water	Expression To Use
Downstream	With Current	Increases Rate	Add Current To Rate	$x + c$
Upstream	Against Current	Slows Down Rate	Deduct Current From Rate	$x - c$

When wind speed is involved, the words "speed of the wind" and "wind blowing" may be stated in the problem, or you may see words such as "tailwind" or "headwind".

When an object is traveling with a tailwind, it means that the object has the wind behind it, and it is being pushed along by the pressure of the wind. Therefore, the object would be traveling at a faster speed.

In turn, an object with a headwind would be traveling against the wind, and would be held back by air resistance. Therefore, the object would be traveling at a slower speed.

The chart below will give you the information you need to name expressions for the rate when the speed of the wind is involved. The variable x represents the rate of an object in still air, without the influence of any wind. The variable w represents the speed of the wind.

Direction Of Wind	**What It Means**	**What It Does**	**If x Is Rate In Still Air**	**Expressio n To Use**
Tailwind	With The Wind	Increases Rate	Add Wind To Rate	$x + w$
Headwind	Against The Wind	Slows Down Rate	Deduct Wind From Rate	$x - w$

12.3 Solving The Problem

Step 1
Read Through The Entire Problem

To fill in your pre-equation chart, you need to look for and take note of distances given, whether or not there is Direct Translation for the rates or there is information about influences on the rates as explained in 12.2. To set up your equation, you need to note if the time given is the same for both objects, or if you are given a total time.
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Step 2
Set Up And Fill In The Chart

Set up a pre-equation chart like the one below. This will be used to determine the expression for Time that you will need for the equation. The formula used is a variation of the distance formula, which is manipulated in order to change it to a formula that will give a rational expression for the time. This variation is Distance divided by Rate equals Time.

$$\textbf{Time }(t) = \frac{\textbf{Distance}(d)}{\textbf{Rate}(r)}$$

D	***r***	***t***

Enter in your chart the information in the problem that refers to distance and its corresponding rate (speed).

- Fill in the Distance Column (d) with each distance given in the problem.
- Name the expressions for the rates as explained in 12.2, and enter them in the Rate Column.
- To get the expressions to enter in the Time Column (t) of the chart, use the variation of the distance formula.
- Write the expression for the time as a rational expression (fraction) with distance over rate.

Step 3

Set Up An Equation

From the chart, you will need the two rational expressions that you named for the time. You will also need the information given about time in the problem. The information stated will be either the amount of the total time, or it will state that the times are the same.

For the Motion Problems in this section, there are only two possible ways to set up an equation. This is determined by the information about time that is given in the problem.

- If the problem states that the times are the same, set the two rational expressions from the Time Column of the chart *equal to each other*.
- If a total amount of time is given,, *add together* the two rational expressions from the Time Column of the chart and set *equal to the total amount of time given.*

Time Given In Problem	**Equation To Use**
The Same	1st Rational Expression = 2nd Rational Expression

As A Total	1st Rational Expression + 2nd Rational Expressions = Total Time

Step 4
Solve the Equation

Using the method taught by your instructor, solve the equation for the variable.

Keep in mind when solving a Motion -- Rational Equations problem, you may get two solutions to the equation. If one of these solutions is negative, eliminate it because a rate (speed) **cannot be negative**.

If both of the solutions are positive, you must eliminate one of them by substituting each of them into the expressions named in Step 2. Whichever solution makes the value of an expression negative will be eliminated.

Step 5
Make Sure to Answer the Question Being Asked

In Motion Problems, as in other word problems, you need to make sure exactly what question is being asked. It is possible that the value for the variable x may be your answer. But it may *not* be.

For example, the value for x may be the rate of an object in still water, and the question may ask for the rate of the object if it is traveling upstream. In this case, you need to substitute the solution for the variable into the expression for an object traveling upstream that you named in Step 2.

EXAMPLES

EXAMPLE 1 When it was raining, Elliott drove for 120 miles. When the rain stopped, he drove 20 mph faster than he did while it was raining. He drove for 300 miles after the rain stopped. If Elliott drove for a total of 10 hours, how fast did he drive while it was raining?

SOLUTION

Step 1 *Read The Problem*

- There are two distances given. One is 120 miles and the other is 300 miles.
- There are Direct Translation words [faster than] in order to determine the rates.
- The rates are x, and $x + 20$. Fill them in next to their corresponding distances.
- The *total* time is given.

Step 2 *Fill In The Chart*

- In your chart, fill in the two distances given in the problem.
- Fill in the two expressions for the rates next to their corresponding distances.
- Fill in the Time by using the rational expression of Distance over Rate.

d	***r***	***t***
120	x	$\frac{120}{x}$
300	$x + 20$	$\frac{300}{x+20}$

Step 3 *Set Up The Equation*

- Use the two rational expressions for the time that you named in your chart in Step 2.
- The total time is given in the problem. It is 10 hours.
- Set up your equation as 1^{st} rational expression + 2^{nd} rational expression = total time.

$$\boxed{\frac{120}{x} + \frac{300}{x+20} = 10}$$

Step 4 *Solve The Equation*

- The solution to the equation is $\boxed{30}$

Step 5 *Answer The Question Asked*

- You have the solution to the equation. The value of x is 30.
- 30 mph is the speed that Elliott drove for 120 miles while it was raining.
- The question asks the speed while raining. You are done. You have the correct answer.

Answer: Elliott drove at a speed of 30 mph while it was raining.

EXAMPLE 2 Melody can cycle for 30 miles against the wind in the same amount of time that it takes her to cycle 66 miles with the wind. If the speed of the wind is 3 mph, what is Melody's speed when she cycles with the wind?

SOLUTION

Step 1 *Read The Problem*

- There are two distances given. One is 30 miles and the other is 66 miles.
- There is no Direct Translation.
- You are given the speed of the wind to determine the rates.
- As per 12.2, the rates are $x - 3$ against the wind, and $x + 3$ with the wind.
- The problem states the amount of time is the same.

Step 2 *Fill In The Chart*

- In your chart, fill in the two distances given in the problem.
- Fill in the two expressions for the rates next to their corresponding distances.
- Fill in the Time by using the rational expression of distance over rate.

Traveling	***d* (distance)**	***r* (rate)**	***t* (time)**
Cycle against the wind	30	$x - 3$	$\frac{30}{x-3}$
Cycle with the wind	66	$x + 3$	$\frac{66}{x+3}$

Step 3 *Set Up The Equation*

- Use the two rational expressions for the time that you named in your chart in Step 2.
- The problem states a total time in the problem. It is 8 hours.
- Set up your equation as 1st rational expression + 2nd rational expression = total time.

$$\frac{30}{x-3} = \frac{66}{x+3}$$

Step 4 *Solve The Equation*

- The solution to the equation is $\boxed{x = 8}$

Step 5 *Answer The Question Asked*

- You have the solution to the equation, but it is not the answer to the question.
- The value of x is 8, which is Melody's rate without the influence of the wind.
- The question asks for Melody's rate with the wind.
- Substitute 8 for x into the expression in your chart for her rate *with* the wind.

$$\text{Rate with wind} = x + 3$$
$$\text{Rate with wind} = 8 + 3$$
$$\text{Rate with wind} = 11$$

Answer: Melody's speed with the wind was 11 mph.

Motion – Rational Equations: Exercise Set

1. Paul can paddle a canoe 15 miles upstream in the same amount of time it takes Liz to paddle a canoe 27 miles downstream. If the current is 2 mph, what is Paul's speed as she travels downstream?

2. A cruise ship traveled for 275 miles with the current in the same amount of time it traveled 175 miles against the current. The speed of the current was 10 mph. What was the speed of the cruise ship while it traveled against the current?

3. Terry can run 15 miles in the same amount of time it takes Mark to run 21 miles. If Mark runs 2 mph faster than Terry, how fast does Mark run?

4. Doni can drive 100 miles to work in the same amount of time that it takes her to drive 60 miles to a friend's house. When driving to her friend's house, Doni drives 20 mph slower than when she drives to work. How fast does she drive going to work?

5. A boat travels 165 miles downstream in the same time the boat travels 135 miles upstream. If the speed of the current is 5 mph, what would be the speed of the boat in still water?

6. A steamboat travels 246 miles upstream in the amount of time the steamboat travels 294 miles downstream. If the speed of the current is 4 mph, how fast would the steamboat travel in still water?

7. The Goodyear blimp flies 153 miles with a tailwind in the same time it travels 57 miles with a headwind. If the speed of the wind is 16 mph, what is the speed of the blimp in still air?

8. A plane flies 960 miles with the wind in the same amount of time that the plane flies 640 miles against the wind. If the speed of the wind is 40 mph, what would the speed of the plane be in still air?

9. A two-engine Cessna flew for 510 miles with a tailwind of 40 mph in the same amount of time that it flew for 330 miles with a headwind of 20 mph. What was the speed of the Cessna when it was traveling with a headwind?

10. On their vacation, a family traveled 135 miles by train and then traveled 855 miles by plane. The speed of the plane was three times the speed of the train. If the total time of the trip was 6 hours, what was the speed of the train?

11. Harriet drove for 90 miles in the city. When she got on the highway she increased her speed by 20 mph and drove for 130 miles. If Harriet drove for a total of 4 hours, how fast did she drive on the highway?

12. Frankie Kowalski drove his jeep for 135 miles on a paved road. He decreased his speed by 25 mph when he went on a dirt rode. He traveled for 40 miles on the dirt rode. If Frankie drove his jeep for 5 hours, what was his speed when driving on the paved road?

13. A homing pigeon can fly 90 miles in the same amount of time an eagle can fly 100 miles. A pigeon flies 5 mph slower than an eagle. How fast can each bird fly?

14. During a marathon, Allen jogged 35 miles. When he got tired, he walked 6 miles to the finish line. Allen jogs 4 mph faster than he walks. If it took him 7 hours to finish the marathon, how fast was he jogging?

15. A hot air balloon travels 40 miles against the wind in the same amount of time that it travels 200 miles with the wind. If the speed of the wind is 20 mph, how fast does the hot air balloon travel against the wind?

16. Tom traveled 85 miles downstream on a raft in the same amount of time he traveled 15 miles upstream on his raft. If the speed of the current was 7 mph , how fast did he travel upstream?

17. Trevor rides a motorcycle 165 miles in the same amount of time he rides a bicycle 45 miles. Trevor can ride his motorcycle 40 mph faster than his bicycle. How fast does he ride his motorcycle?

18. A passenger train can travel 280 miles in the same amount of time a freight train can travel 140 miles. The passenger train travels 35 mph faster than the freight train. How fast does the passenger train travel?

19. On an island excursion, some tourists walked 7 miles on a nature path and then hiked 12 miles up a mountainside. The tourists walked 3 mph faster than they hiked. The total time of the excursion was 4 hours. At what rate did the tourists hike?

20. Karl is training for a triathlon. He trains for the swimming portion at the local beach One day Karl swam for a total of 6 hours. He swam 3 miles against the current, and 33 miles with the current. That day the speed of the current was 5 miles per hour. How fast was Karl swimming with the current?