Determining Sample Size

For

Confidence Interval

Sample Size for **Proportion**:

- We know the margin of error is $E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$ when constructing confidence interval for population proportion.

- With some algebra work, we can get: $n = \hat{p} \cdot \hat{q} \cdot \left(\frac{Z_{\alpha/2}}{E} \right)^2$

Always round up your final answer.

Proportion Sample Size Chart:

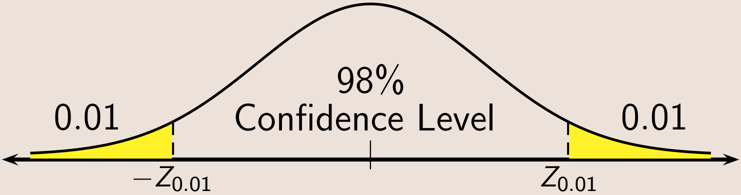
When	Use
$\hat{p}$ or $\hat{q}$ are known	$n = \hat{p} \cdot \hat{q} \cdot \left(\frac{Z_{\alpha/2}}{E}\right)^2$
$\hat{p}$ and $\hat{q}$ are unknown Assume $\hat{p} = 0.5$ and $\hat{q} = 0.5$	$n = 0.25 \cdot \left(\frac{Z_{\alpha/2}}{E}\right)^2$

Example:

Find the minimum sample size needed if we wish to construct 98% confidence interval for population proportion and margin of error not to exceed 5% assuming the sample proportion is $\hat{p} = 0.35$.

Solution:

With our desired confidence level of 98%,
we first find the critical value,



$Z_{0.01} = \textbf{invNorm}(0.99, 0, 1) = 2.326$

Solution Continued:

We are also given $\hat{p} = 0.35$, we can find $\hat{q}$.

$\hat{q} = 1 - \hat{p} = 1 - 0.35 = 0.65$

With the margin of error not to exceed 5%, we have $E = 0.05$, we are now ready to use the formula to determine the minimum sample size.

$$n = \hat{p} \cdot \hat{q} \cdot \left(\frac{Z_{\alpha/2}}{E}\right)^2 = 0.35 \cdot 0.65 \cdot \left(\frac{2.326}{0.05}\right)^2 \approx 492.335$$

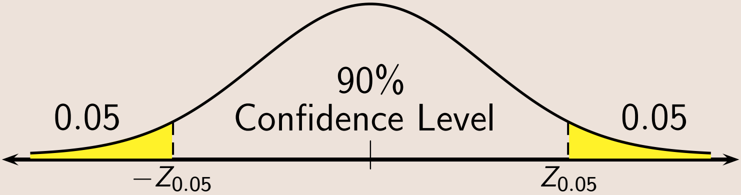
we always round up your final answer, so the minimum sample size is 493.

Example:

Find the minimum sample size needed if we wish to construct 90% confidence interval for population proportion and margin of error not to exceed 4% assuming $\hat{p}$ & $\hat{q}$ are unknown.

Solution:

With our desired confidence level of 90%,
we first find the critical value,



$Z_{0.05} = \textbf{invNorm}(0.95, 0, 1) = 1.645$

Solution Continued:

We are told to assume that $\hat{p}$ and $\hat{q}$ are both unknown,

When $\hat{p}$ and $\hat{q}$ are unknown, we use 0.5 for both.

With the margin of error not to exceed 4%, we have $E = 0.04$, we are now ready to use the formula to determine the minimum sample size.

$$n = 0.25 \cdot \left(\frac{Z_{\alpha/2}}{E}\right)^2 = 0.25 \cdot \left(\frac{1.645}{0.04}\right)^2 \approx 422.816$$

we always round up your final answer, so the minimum sample size is 423.

Sample Size for **Mean**:

- We know the margin of error is $E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ when constructing confidence interval for population mean.
- With some algebra work, we can get: $n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$

Always round up your final answer.

Mean Sample Size Chart:

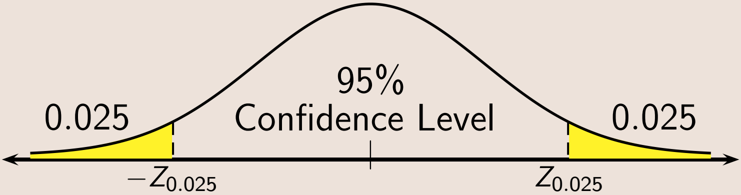
When	Use
σ is known	$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$
σ is unknown	$n = \left(\frac{Z_{\alpha/2} \cdot s}{E}\right)^2$

Example:

Find the minimum sample size needed if we wish to construct 95% confidence interval for population mean and margin of error not to exceed 10 given the population standard deviation is 25.

Solution:

With our desired confidence level of 95%,
we first find the critical value,



$Z_{0.025} = \textbf{invNorm}(0.975, 0, 1) = 1.960$

Solution Continued:

We are also given $\sigma = 25$, with the margin of error 10.

We are now ready to use the formula to determine the minimum sample size.

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{1.960 \cdot 25}{10}\right)^2 \approx 24.01$$

We always round up your final answer,

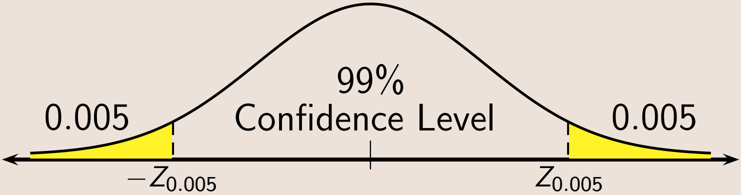
So the minimum sample size is 25.

Example:

Find the minimum sample size needed if we wish to construct 99% confidence interval for population mean and margin of error not to exceed 8 assuming a sample standard deviation 12.5.

Solution:

With our desired confidence level of 99%,
we first find the critical value,



$Z_{0.005} = \textbf{invNorm}(0.995, 0, 1) = 2.576$

Solution Continued:

We are also given $s = 12.5$, with the margin of error 8.

We are now ready to use the formula to determine the minimum sample size.

$$n = \left(\frac{Z_{\alpha/2} \cdot s}{E}\right)^2 = \left(\frac{2.576 \cdot 12.5}{8}\right)^2 \approx 16.201$$

We always round up your final answer,

So the minimum sample size is 17.