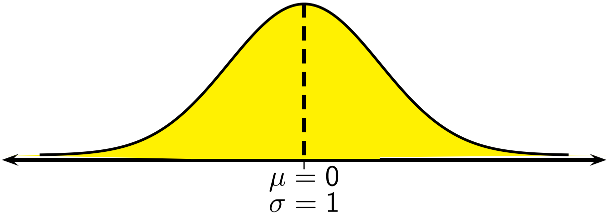
Standard Normal

Probability Distribution

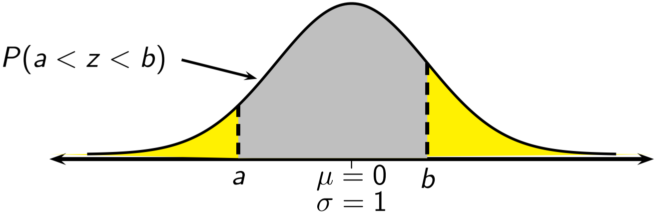
What is a **Standard Normal Distribution**?

It is a probability distribution for a continuous random variable z that can assume any values such that

- Mean, mode, and median are all equal,
- $\mu = 0$ and $\sigma = 1$,
- The graph of the distribution is symmetric, and bell-shaped,



- The total area under the curve is equal to 1,
- $P(z = c) = 0$, and
- $P(a < z < b)$ is the area under the curve on the interval (a, b).



- Commonly used notation: $N(0, 1)$

Standard Normal Distribution & TI

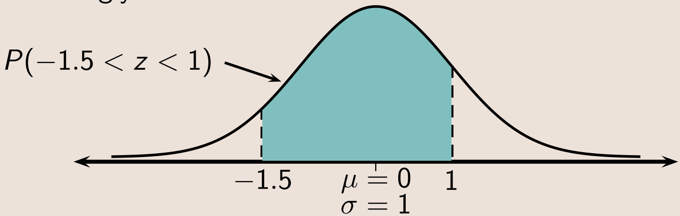
Standard Normal Distribution	TI Command
$P(a < z < b)$	normalcdf$(a, b, 0, 1)$
$P(z < a)$	normalcdf$(-E99, a, 0, 1)$
$P(z > b)$	normalcdf$(b, E99, 0, 1)$
$P(z < a \text{ or } z > b)$	1- normalcdf$(a, b, 0, 1)$

Example:

Find $P(-1.5 < z < 1)$.

Solution:

We start by drawing the normal curve, then shade and label accordingly.



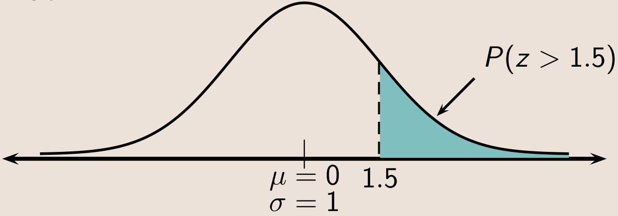
Now we can use the normal distribution chart to compute the area which is the probability that we are looking for. In this case, $P(-1.5 < z < 1) = normalcdf(-1.5, 1, 0, 1) = 0.775$.

Example:

Find $P(z > 1.5)$.

Solution:

We start by drawing the normal curve, then shade and label accordingly.



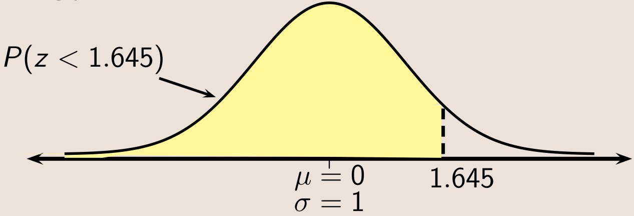
Now we can use the normal distribution chart to compute the area which is the probability that we are looking for. In this case, $P(z > 1.5) = normalcdf(1.5, E99, 0, 1) = 0.067$.

Example:

Find $P(z < 1.645)$.

Solution:

We start by drawing the normal curve, then shade and label accordingly.



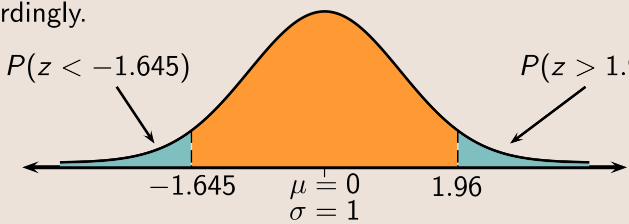
Now we can use the normal distribution chart to compute the area which is the probability that we are looking for. In this case, $P(z < 1.645) = normalcdf(-E99, 1.645, 0, 1) = 0.950$.

Example:

Find $P(z < -1.645 \text{ or } z > 1.96)$.

Solution:

We start by drawing the normal curve, then shade and label accordingly.

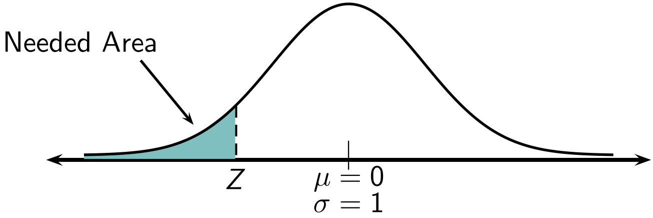


Using the normal distribution chart to compute
$P(z < -1.645 \text{ or } z > 1.96) = 1 - P(-1.645 < z < 1.96) = 0.075.$

How do we find Z **Score** from **Known Area**?

It is a probability distribution for a continuous random variable z that can assume any values such that

- Draw a bell-shaped curve,
- Clearly identify and shade the region that represents the known area,



Working with the cumulative area from the left, use the normal distribution chart to find the corresponding Z score.

Standard Normal Distribution & TI

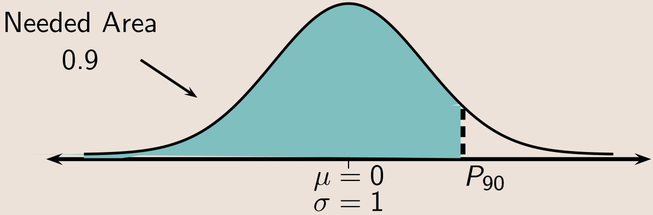
Standard Normal Distribution	TI Command
$Z = P_k$	$\text{invNorm}(k\%, 0, 1)$

Example:

Find $Z = P_{90}$.

Solution:

We know that P_{90} separates the bottom 90% from the top 10%, so the left area is 0.9.



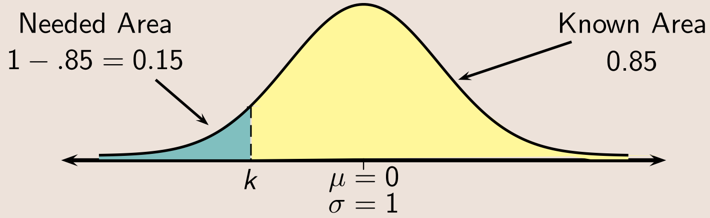
So we get $Z = P_{90} = invNorm(0.9, 0, 1) = 1.282$.

Example:

Find k such that $P(z > k) = 0.85$.

Solution:

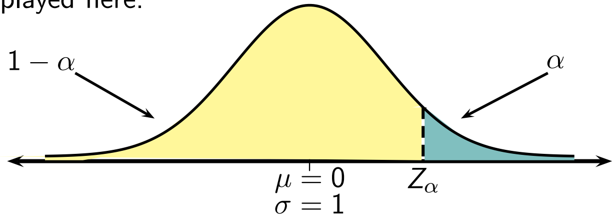
We are given $P(z > k) = 0.85$ which implies the area to the right of k is 0.85 but we need the area to the left of k.



Now we get $k = invNorm(0.15, 0, 1) = -1.036$.

What is Z_α?

It is a notation that describes a z score with an area α to its right as displayed here.



How to find Z_α with **TI**:

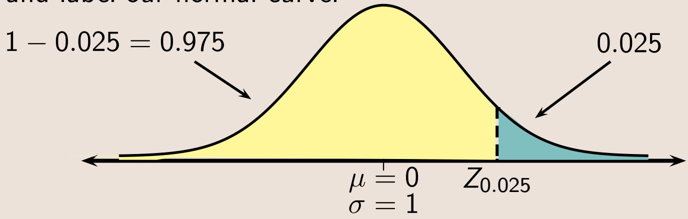
Standard Normal Distribution	TI Command
Z_α	invNorm$(1-\alpha, 0, 1)$

Example:

Find $Z_{0.025}$.

Solution:

Given $Z_{0.025}$, we have $\alpha = 0.025$, and $1 - \alpha = 0.975$, so we draw and label our normal curve.



Now we get $Z_{0.025} = \mathit{invNorm}(0.975, 0, 1) = 1.960$.