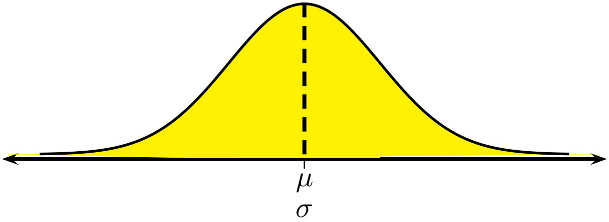
Normal Probability Distribution

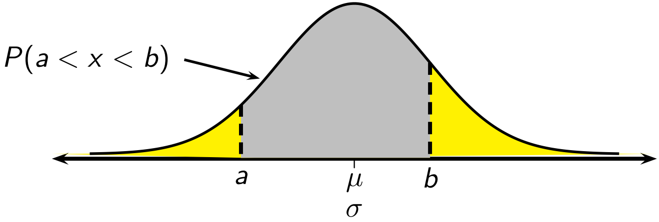
What is a **Normal Probability Distribution**?

It is a probability distribution for a continuous random variable x that can assume any values such that

- Mean, mode, and median are all equal,
- μ and σ are given in the problem,
- The graph of the distribution is symmetric, and bell-shaped,



- The total area under the curve is equal to 1,
- $P(x = c) = 0$, and
- $P(a < x < b)$ is the area under the curve on the interval (a, b).



- Commonly used notation: $N(\mu, \sigma)$

Normal Probability Distribution & TI

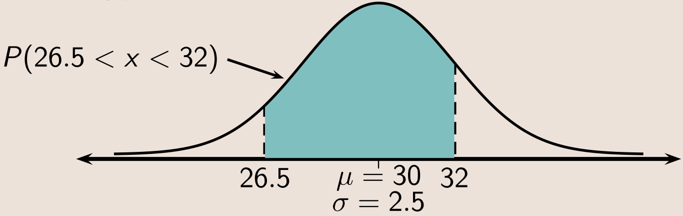
Normal Probability Distribution	TI Command
$P(a < x < b)$	normalcdf(a, b, μ, σ)
$P(x < a)$	normalcdf$(-E99, a, \mu, \sigma)$
$P(x > b)$	normalcdf$(b, E99, \mu, \sigma)$
$P(x < a \text{ or } x > b)$	1- normalcdf(a, b, μ, σ)

Example:

Find $P(26.5 < x < 32)$ given $N(30, 2.5)$.

Solution:

We start by drawing the normal curve, then shade and label accordingly.



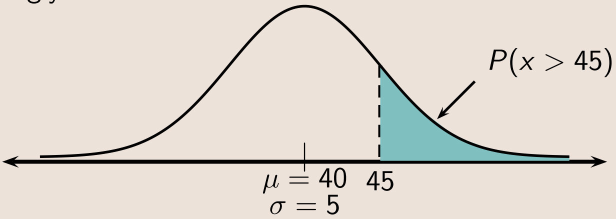
Now we can use the normal distribution chart to compute the area which is the probability that we are looking for. In this case, $P(26.5 < x < 32) = \mathit{normalcdf}(26.5, 32, 30, 2.5) = 0.707$.

Example:

Find $P(x > 45)$, rounded to the nearest whole percent given $N(40, 5)$.

Solution:

We start by drawing the normal curve, then shade and label accordingly.



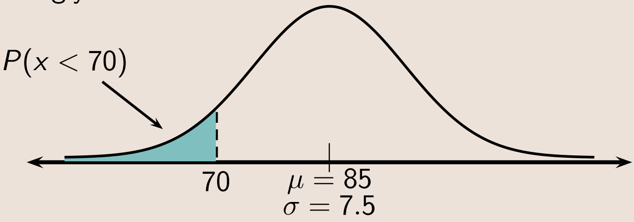
Now we can use the normal distribution chart to compute the area which is the probability that we are looking for. In this case, $P(x > 45) = \mathit{normalcdf}(45, E99, 40, 5) = 0.159 \approx 16\%$.

Example:

Find $P(x < 70)$, rounded to three decimal places given $N(85, 7.5)$.

Solution:

We start by drawing the normal curve, then shade and label accordingly.



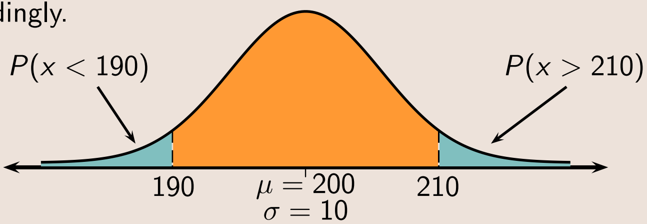
Now we can use the normal distribution chart to compute the area which is the probability that we are looking for. In this case, $P(x < 70) = \mathit{normalcdf}\,(-E99, 70, 85, 7.5) = 0.023$.

Example:

Find $P(x < 190 \text{ or } x > 210)$ given $N(200, 10)$.

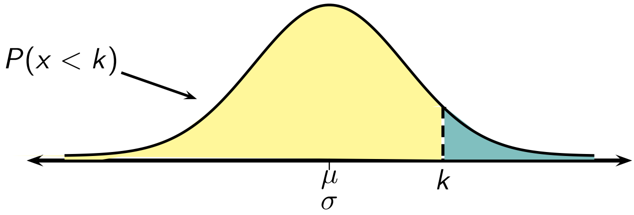
Solution:

We start by drawing the normal curve, then shade and label accordingly.



Using the normal distribution chart to compute
$P(x < 190 \text{ or } x > 210) = 1 - P(190 < x < 210) \approx 0.317$.

Finding k when $P(x < k)$ is given with **TI**:



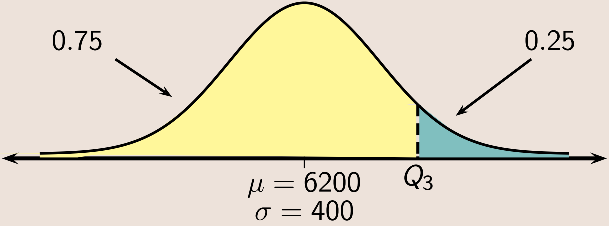
Standard Normal Distribution	TI Command
k	invNorm$(P(x < k), \mu, \sigma)$

Example:

Find $x = Q_3$, rounded to a whole number, given $N(6200, 400)$.

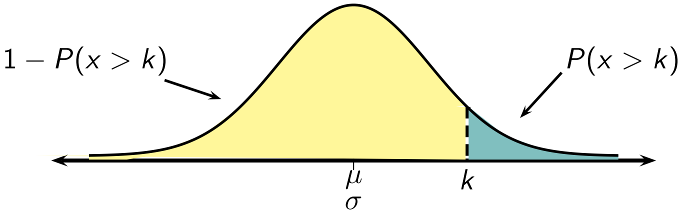
Solution:

We know $x = Q_3$ simply means find k such that $P(x < k) = 0.75$ which separates the bottom 75% from the top 25%, so we draw and label our normal curve.



Now we get $x = Q_3 = invNorm(0.75, 6200, 400) \approx 6470$.

Finding k when $P(x > k)$ is given with **TI**:



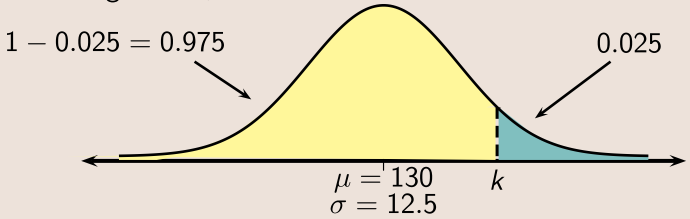
Standard Normal Distribution	TI Command
k	invNorm$(1 - P(x > k), \mu, \sigma)$

Example:

Find k such that $P(x > k) = 2.5\%$ with $N(130, 12.5)$.

Solution:

We convert 2.5% to decimal notation 0.025, and that is the area to the right of k, so we draw and label our normal curve.



Now we get $k = invNorm(0.975, 130, 12.5) \approx 154.5$.