Geometric Probability Distribution

What is a **Geometric Probability Distribution**?

It is a probability distribution for a discrete random variable x with probability $P(x)$ such that

- The same criteria as binomial probability distribution exists except the number of trials n is not a fixed number.
- A trial is repeated until a success occurs on the x trial where $x \geq 1$.
- The repeated trials are independent of each other.
- The probability of success p remains the same for each trial where $0 \leq p \leq 1$.
- $0 \leq P(x) \leq 1$ and $\sum P(x) = 1$.

How to find the probability of a

Geometric Probability Distribution:

The **probability that the first success will occur on trial number** x is

$$P(x) \;=\; p \cdot (q)^{x-1}, \text{ where } q = 1 - p$$

with $\mu = \dfrac{1}{p}$, $\sigma^2 = \dfrac{q}{p^2}$, and $\sigma = \sqrt{\sigma^2}$.

Example:

Consider a geometric probability distribution for a discrete random variable x with probability of success $p = .2$.

- Find q.
- Find $P(x = 2)$.
- Find $P(x \leq 2)$.
- Find its mean μ.
- Find its variance σ^2.
- Find its standard deviation σ.

Solution:

- Find $q \Rightarrow q = 1 - p = 1 - 0.2 = 0.8$.
- Find $P(x = 2) \Rightarrow P(x = 2) = 0.2 \cdot (0.8)^{2-1} = 0.16$.
- Find $P(x \leq 2) \Rightarrow P(x \leq 2) = P(x = 2) + P(x = 1) = 0.16 + 0.2 = 0.36$.
- Find its mean $\mu \Rightarrow \mu = \dfrac{1}{p} = \dfrac{1}{.2} = 5$.
- Find its variance $\sigma^2 \Rightarrow \sigma^2 = \dfrac{q}{p^2} = \dfrac{.8}{.2^2} = 20$.
- Find its standard deviation $\sigma \Rightarrow \sigma = \sqrt{\sigma^2} = \sqrt{20} \approx 4.5$.

Geometric Probability Distributions & TI

When you have	Use TI command
$P(x = a)$	geometpdf(p, a)
$P(x \leq a)$	geometcdf(p, a)
$P(x \geq a)$	$1 -$ geometcdf$(p, a - 1)$
$P(a \leq x \leq b)$	geometcdf$(p, b) -$ geometcdf$(p, a - 1)$

You can find TI commands **geometpdf** and **geometcdf** by pressing 2ND, VARS, then $\downarrow$ to locate them.

Example:

Certain basketball player in NBA makes 70% of his free throws. What is the probability that

- he misses his first two free throws and makes the third one.
- he makes his first or second free throws.
- he makes his first free throw after the fourth attempt.

Solution:

This problem fits all criteria of a geometric probability distribution with $p = 0.7$ and $q = 0.3$.

Let x be the number of free throws when the first success occurs.

Solution Continued:

Now we need to find the probability that

- he misses his first two free throws and makes the third one
  $\Rightarrow P(x=3) = \text{geometpdf}(.7, 3) = 0.063.$

- he makes his first or second free throws
  $\Rightarrow P(x \leq 2) = \text{geometcdf}(.7, 2) = 0.91.$

- he makes his first free throw after the fourth attempt
  $\Rightarrow P(x > 4) = P(x \geq 5) = 1 - P(x \leq 4)$
  $\Rightarrow 1 - \text{geometcdf}(.7, 4) = 0.0081.$

It is important to emphasize that we cannot do the last part by computing $P(x \leq 4)$ by switching p and q, so using $p = .3$ we will not get the correct answer.

$\Rightarrow P(x \leq 4) = \textbf{geometcdf}(.3, 4) \neq 0.0081.$