Expected Value

What is **Expected Value**?

Expected Value is a measurement of the mean of a probability distribution.

How do we compute **Expected Value**?

If x is a discrete random variable with a discrete probability distribution $P(x)$, then the expected value is defined by

$$\begin{aligned} E(x) &= \mu \\ &= \sum x \cdot P(x) \end{aligned}$$

Finding Expected Value Using TI:

- Organize the problem in a table.

Net Gain	P(Net Gain)

- Clear and reset all lists.
- Enter Net Gain in L1, and corresponding P(net Gain) in L2.
- Perform basic computation using L1 and L2.
- **Expected Value** is the value of $\bar{x}$.

Example:

At a college fundraising event, the math club sold 400 tickets for \$5 each. The winning ticket will receive a brand new calculator valued at \$100. What is the expected value for net earning per ticket for the math club?

Solution:

Since there is only one winning ticket, then

$$P(Win) = \frac{1}{400} \text{ and } P(\overline{Win}) = \frac{399}{400}.$$

The amount of net gain for the winning ticket is $\$100 - \$5 = \$95$,

and the rest of the tickets each have a \$5 net loss.

Solution Continued:

Now we can make our table.

Net Gain	P(Net Gain)
95	$\frac{1}{400}$
-5	$\frac{399}{400}$

Now using $L1$ for Net Gain and $L2$ for P(net Gain), now we can find the expected value.

The expected value for the math club is \$4.75 per ticket.

Example:

At a game, you can place a bet for \$6 and draw a card randomly from a full deck of playing card. In the event that you draw a face card, the house will give you \$26, otherwise you lose your bet amount of \$6. What is the expected value of this game for the house per bet?

Solution:

Since there are 12 face cards in a full deck of playing cards, then $P(\text{Face Card}) = \frac{12}{52}$ and $P(\overline{\text{Face Card}}) = \frac{40}{52}$.

The amount of net winning for any winning draw is

$\$26 - \$6 = \$20,$

and any other draw has a net loss of \$6.

Solution Continued:

Now we can make our table.

Net Gain	P(Net Gain)
20	$\frac{12}{52}$
-6	$\frac{40}{52}$

Now using $L1$ for Net Gain and $L2$ for P(net Gain), we can find the expected value.

The expected value for the house is \$0 per bet.

Example:

An insurance company sells a one-year term life insurance policy to Mrs. Young for a premium of \$1000. If she dies within one year, the company will pay \$150,000 to her beneficiary. Assume the probability that she will be alive one year later is 99.5%, find the expected value for the insurance company per policy.

Solution:

We are given that
$P(\text{She will be alive}) = 0.995$ and $P(\text{She will not be alive}) = .005$.

If she is not alive, her beneficiary will receive
$\$150000 - \$1000 = \$149000$,

and if she is still alive, she would lose her premium of \$1000.

Solution Continued:

Now we can make our table.

Net Gain	P(Net Gain)
149000	0.005
-1000	0.995

Now using $L1$ for Net Gain and $L2$ for P(net Gain), we can find the expected value.

The expected value for the insurance company is $250 per policy.