Discrete Random Variables

&

Probability Distributions

What is a **Random Variable**?

It is a quantity whose values are real numbers and are determined by the number of desired outcomes of an experiment.

Example:

List the sample space of all outcomes for a family with two children, and the corresponding values of the number of boys.

Solution:

Sample Space	Number of Boys
BB	2
BG	1
GB	1
GG	0

Is there any special **Random Variables**?

We can categorize random variables in two groups:

- Discrete random variable
- Continuous random variable

What are **Discrete Random Variables**?

It is a numerical value associated with the desired outcomes and has either a finite number of values or infinitely many values but countable such as whole numbers $0, 1, 2, 3, \cdots$.

What are **Continuous Random Variables**?

It has infinite numerical values associated with any interval on the number line system without any gaps or breaks.

Example:

Classify the following random variables as discrete or continuous.

1. The number of accidents on the 60 freeway.
2. The length of time it takes to play a baseball game.
3. The amount of milk produced by a cow in a month.
4. The number of correct answers on a multiple-choice exam.

Solution:

Classify the following random variables as discrete or continuous.

1. The number of accidents. $\Rightarrow$ Discrete
2. The length of time. $\Rightarrow$ Continuous
3. The amount of milk. $\Rightarrow$ Continuous
4. The number of correct answers. $\Rightarrow$ Discrete

What is a **Probability Distribution**?

It is a description and often given in the form of a graph, formula, or table, that provides the probability for all possible desired outcomes of the random variable.

Are there any **Requirements**?

Let x be any random variable and $P(x)$ be the probability of the random variable x, then

- $\sum P(x) = 1$
- $0 \leq P(x) \leq 1$

Example:

Let x be the number of defective items when 2 items have shipped and $P(x)$ be probability of x defective items.Use the table below to find $P(x = 2)$.

x	0	1	2
$P(x)$	$\frac{2}{5}$	$\frac{8}{15}$	

Solution:

From the table, we have $P(x=0)=\frac{2}{5}$, $P(x=1)=\frac{8}{15}$ and we also know that $\sum P(x)=1$. So

$$\begin{aligned}
\sum P(x) &= 1\\
P(x=0)+P(x=1)+P(x=2) &= 1\\
\frac{2}{5}+\frac{8}{15}+P(x=2) &= 1\\
\frac{14}{15}+P(x=2) &= 1\\
P(x=2) &= 1-\frac{14}{15}\\
P(x=2) &= \frac{1}{15}
\end{aligned}$$

What is a **Discrete Probability Distribution**?

It is a probability distribution for a discrete random variable x with probability $P(x)$ such that

- $\sum P(x) = 1$, and
- $0 \leq P(x) \leq 1$.

How to Draw **Probability Distribution Histogram**:

- Use x values like midpoints on the horizontal axis,
- Use $P(x)$ for the height of each bar of the histogram.

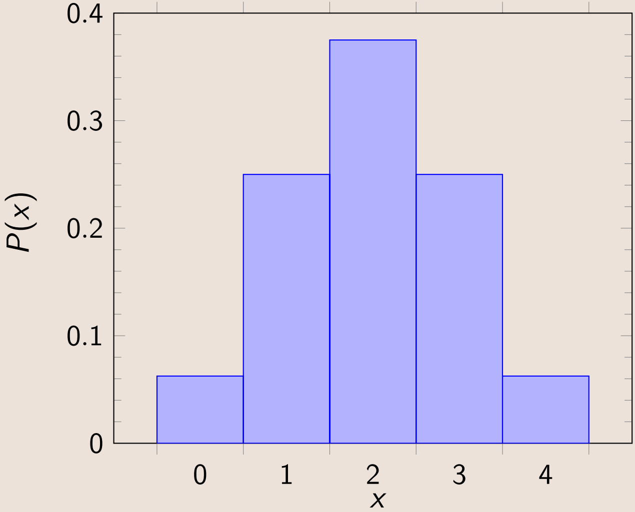
Example:

Let x be the number of boys in a family with 4 children and $P(x)$ be the probability of the number of boys as shown in the table here.

x	$P(x)$
0	.0625
1	.2500
2	.3750
3	.2500
4	.0625

Use the table above to draw the probability distribution histogram for the number of boys in a family with 4 children.

Solution:



Finding **Mean**, **Variance**, and **Standard Deviation**

of **Discrete Probability Distribution**

Given a probability distribution for a discrete random variable x with probability $P(x)$, then

- Mean $\Rightarrow \mu = \sum \left[x \cdot P(x)\right]$,
- Variance $\Rightarrow \sigma^2 = \sum \left[x^2 \cdot P(x)\right] - \mu^2$, and
- Standard Deviation $\Rightarrow \sigma = \sqrt{\sigma^2}$.

Example:

Let x be the number of boys in a family with 4 children and $P(x)$ be the probability of the number of boys as shown in the table here.

x	$P(x)$
0	.0625
1	.2500
2	.3750
3	.2500
4	.0625

Use the table above to find the mean, variance, and standard deviation of the number of boys in a family with 4 children.

Solution:

We begin by extending our table and compute $\sum x \cdot P(x)$, and $\sum x^2 \cdot P(x)$.

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	.0625	$0 \cdot .0625$	$0^2 \cdot .0625$
1	.2500	$1 \cdot .2500$	$1^2 \cdot .2500$
2	.3750	$2 \cdot .3750$	$2^2 \cdot .3750$
3	.2500	$3 \cdot .2500$	$3^2 \cdot .2500$
4	.0625	$4 \cdot .0625$	$4^2 \cdot .0625$
Total	1	2	5

Now Using the formula, we get $\mu = 2$, $\sigma^2 = 1$, and $\sigma = 1$.

---

we can use technology to find these values as these calculations have a great tendency to get nasty quickly.

Example:

In a survey of 250 randomly selected registered students in a summer session, 35 students were taking 3 units, 75 students were taking 4 units, 95 students were taking 5 units, and the rest were taking 6 units. Complete the following table.

Number of units	Probability of the Number of Units

and then find the mean, variance, and standard deviation of the number of units students are taking in a summer session.

Solution:

We will first compute the probabilities from the information in the survey. Let x be the number of units taken by a student so $P(x=3)=\dfrac{35}{250}=0.14$, $P(x=4)=\dfrac{75}{250}=0.30$, and so on.

We are now ready to complete the table.

x(number of units)	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
3	0.14	$3 \cdot 0.14$	$3^2 \cdot 0.14$
4	0.30	$4 \cdot 0.30$	$4^2 \cdot 0.30$
5	0.38	$5 \cdot 0.38$	$5^2 \cdot 0.30$
6	0.18	$6 \cdot 0.18$	$6^2 \cdot 0.18$
Total	1	4.6	22.04

Now using the formulas, we get $\mu = 4.6$, $\sigma^2 = 0.88$, and $\sigma = 0.938$.

Discrete Probability Distributions & TI

- Organize the given information in a table in two columns with heading x and $P(x)$.
- Enter all x values in L_1.
- Enter all $P(x)$ values in L_2.
- Execute the following command as they fit your type of calculator to find $\mu = \bar{x}$ and $\sigma = \sigma_x$.