Binomial Probability Distribution

What is **Binomial Probability Distribution**?

It is a probability distribution for a discrete random variable x with probability $P(x)$ such that

- $\sum P(x) = 1$.
- $0 \leq P(x) \leq 1$.
- It has a fixed number of independent events.
- Each event has only two outcomes, and are referred to as success and failure.
- The probability of success and failure remains the same for all events.

Binomial Probability Distribution Notations:

- Number of independent trials $\Rightarrow n$
- Number of successes $\Rightarrow x$
- Probability of success in one of the trials $\Rightarrow p$
- Probability of failure in one of the trials $\Rightarrow q$ where $p + q = 1$

Binomial Probability Distributions Formula:

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x} \text{ for } x = 0, 1, 2, 3, \cdots, n$$

Example:

Given: Binomial probability distribution with $n = 25$, and $p = .8$. Find $P(x = 18)$.

Solution:

We first find
$q = 1 - p = 1 - 0.8 = 0.2,$
Now we can use the binomial probability distribution formula to find $P(x = 18)$.

$$\begin{aligned} P(x = 18) &= {}_{25}C_{18} \cdot (.8)^{18} \cdot (.2)^{25-18} \\ &= 480700 \cdot (.8)^{18} \cdot (.2)^{7} \\ &\approx 0.1108 \end{aligned}$$

Example:

Given: Binomial probability distribution with $n = 100$, and $p = .5$. Find $P(x = 45 \text{ or } x = 46)$.

Solution:

We first realize that

$$P(x = 45 \text{ or } x = 46) \quad = \quad P(x = 45) + P(x = 46)$$

Now we can use the binomial probability distribution formula to find $P(x = 45)$, and $P(x = 46)$ given the fact that $q = 1 - p = .5$.

$$\begin{aligned} P(x = 45) &= {}_{100}C_{45} \cdot (.5)^{45} \cdot (.5)^{55} \\ &\approx 0.0485 \end{aligned}$$

and

Solution Continued:

$$\begin{aligned} P(x = 46) &= {}_{100}C_{46} \cdot (.5)^{46} \cdot (.5)^{54} \\ &\approx 0.0580 \end{aligned}$$

Now we are ready to proceed

$$\begin{aligned} P(x = 45 \text{ or } x = 46) &= P(x = 45) + P(x = 46) \\ &\approx 0.0485 + 0.0580 \\ &\approx 0.1065 \end{aligned}$$

Example:

Given: Binomial probability distribution with $n = 20$, and $p = .2$.
Find $P($ at least one success $)$.

Solution:

The key word here is at least one, we know that

$$P(\text{at least one}) = 1 - P(\text{none})$$

And if we let x to be the number of successes, then we have

$$P(x \geq 1) = 1 - P(x = 0)$$

So with $n = 20$, and $p = .2$, we have

$$\begin{aligned} P(x \geq 1) &= 1 -_{20} C_0 \cdot (.2)^0 \cdot (.8)^{20} \\ &\approx 1 - 0.0115 \\ &\approx 0.9885 \end{aligned}$$

Binomial Probability Distributions Keywords

Keyword	Translation
exactly	$P(x = a)$
at most	$P(x \leq a)$
at least	$P(x \geq a)$
fewer than, below	$P(x < a)$
more than, exceed	$P(x > a)$
between a and b, inclusive	$P(a \leq x \leq b)$
from a to b	$P(a \leq x \leq b)$

When working with $P(x < a)$ or $P(x > a)$, do the following:

- $P(x < a) \Rightarrow P(x \leq a - 1)$
- $P(x > a) \Rightarrow P(x \geq a + 1)$

Example:

You are about to take a multiple-choice test with 20 questions. Each question has 5 possible choices but only one choice is correct. If you were to guess randomly on all questions,

1. what is the probability of guessing exactly 3 correct answers?
2. what is the probability of guessing fewer than 4 correct answers?
3. what is the probability of guessing at least 3 correct answers?

Solution:

We first obtain from the problem that $n = 20$, $p = \frac{1}{5} = 0.2$, and $q = \frac{4}{5} = 0.8$. Now

Solution Continued:

1. what is the probability of guessing exactly 3 correct answers? $\Rightarrow P(x=3)$, so

$$\begin{aligned} P(x=3) &= {}_{20}C_3 \cdot (.2)^3 \cdot (.8)^{17} \\ &\approx 0.205 \end{aligned}$$

2. what is the probability of guessing fewer than 4 correct answers? $\Rightarrow P(x<4)=P(x\leq 3)$, so

$$\begin{aligned} P(x\leq 3) &= P(x=0)+\cdots+P(x=3) \\ &\approx 0.411 \end{aligned}$$

3. what is the probability of guessing at least 3 correct answers? $\Rightarrow P(x\geq 3)$, so

$$\begin{aligned} P(x\geq 3) &= 1-P(x\leq 2) \\ &= 1-[P(x=0)+P(x=1)+P(x=2)] \\ &\approx 0.794 \end{aligned}$$

Example:

An airline claims that 96% of passengers with tickets actually show up for the flight. The airline overbooked a flight by selling 145 tickets for the aircraft that has 140 seats. What is the probability that the number of passengers with tickets who show up is between 135 and 140, inclusive?

Solution:

We first obtain from the problem that $n = 145$, $p = 0.96$, and $q = 0.04$. Since we want the probability that the number of passengers with tickets show up is between 135 and 140, inclusive, this translates to $P(135 \leq x \leq 140)$.

$$\begin{aligned} P(135 \leq x \leq 140) &= P(x = 135) + \cdots + P(x = 140) \\ &\approx 0.661 \end{aligned}$$

Finding Binomial Probability Using TI:

- Organize the problem in a table.

Given Information
n
p
x

- Press 2nd, followed by VARS.
- Arrow down several times to find binompdf, or binomcdf.
- Enter the values for trials n, probability p, and number of successes x to complete the following commands.
- binompdf(n, p, x) or binomcdf(n, p, x),
- followed by ENTER.

Binomial Probability Distributions & TI

Binomial Probability	TI Command
$P(x = a)$	binompdf(n, p, a)
$P(x \leq a)$	binomcdf(n, p, a)
$P(x \geq a)$	1−binomcdf$(n, p, a - 1)$
$P(a \leq x \leq b)$	binomcdf(n, p, b)−binomcdf$(n, p, a - 1)$

When working with $P(x < a)$ or $P(x > a)$, do the following:

- $P(x < a) \Rightarrow P(x \leq a - 1)$
- $P(x > a) \Rightarrow P(x \geq a + 1)$

Example:

Given: Binomial probability distribution with $n = 25$, and $p = .8$.
Find $P(x = 18)$.

Solution:

Using TI calculator to find $P(x = 18)$, we get

$$\begin{aligned} P(x = 18) &= \textit{binompdf}(25, .8, 18) \\ &\approx 0.1108 \end{aligned}$$

Example:

Probability of a getting tails when a loaded coin is tossed is 0.6.
Suppose you toss this coin 100 times, find the probability of getting at most 65 tails.

Solution:

The keyword here is at most, which translates to $\leq$, so if we let x to be the number of tails, then

$$P(\text{getting at most 65 tails}) = P(x \leq 65)$$

Using TI calculator to find $P(x \leq 65)$, we get

$$\begin{aligned} P(x \leq 65) &= \textit{binomcdf}\,(100, .6, 65) \\ &\approx .8697 \end{aligned}$$

Example:

You are about to take a multiple-choice test with 40 questions. Each question has 4 possible choices but only one choice is correct. If you were to guess randomly on all questions,

- what is the probability of guessing fewer than 15 correct answers?
- what is the probability of guessing more than 10 correct answers?
- what is the probability of guessing between 8 to 12, inclusive, correct answers?
- what is the probability of guessing fewer than 8 or more than 12 correct answers?

Solution:

We first obtain from the problem that $n = 40$, $p = \frac{1}{4} = 0.25$. Now

- what is the probability of guessing fewer than 15 correct answers? $\Rightarrow P(x < 15)$, so

$$\begin{aligned} P(x < 15) &= P(x \leq 14) \\ &= \mathit{binomcdf}(40, .25, 14) \\ &\approx .9456 \end{aligned}$$

- what is the probability of guessing more than 10 correct answers? $\Rightarrow P(x > 10)$,so

$$\begin{aligned} P(x > 10) &= P(x \geq 11) \\ &= 1 - P(x \leq 10) \\ &= 1 - \mathit{binomcdf}(40, .25, 10) \\ &\approx .4161 \end{aligned}$$

Solution Continued:

- what is the probability of guessing between 8 to 12, inclusive, correct answers? $\Rightarrow P(8 \leq x \leq 12)$, so

  $P(8 \leq x \leq 12) =$

  $\mathit{binomcdf}(40, .25, 12) - \mathit{binomcdf}(40, .25, 7) \approx .6389$

- what is the probability of guessing fewer than 8 or more than 12 correct answers? $\Rightarrow P(x < 8 \text{ or } x > 12)$,so

$$\begin{aligned} P(x < 8 \text{ or } x > 12) &= 1 - P(8 \leq x \leq 12) \\ &= 1 - .6389 \\ &\approx .3611 \end{aligned}$$