Addition Rule in Probability

with

Venn Diagram

What is the **Addition Rule**?

Given two different events A and B, the **Addition Rule** states

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Where $P(A \text{ and } B)$ implies the probability that both events occur at the same time as an outcome.

Example:

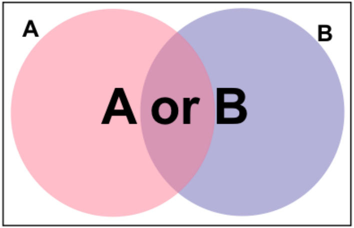
Given $P(A) = 0.65, P(B) = 0.45,$ and $P(A \text{ and } B) = 0.35,$ find $P(A \text{ or } B)$.

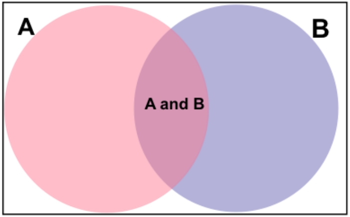
Solution:

Using the addition rule, we get

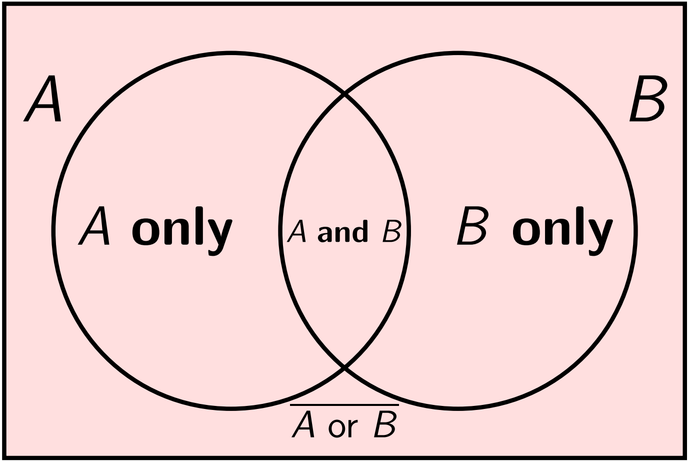
$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= 0.65 + 0.45 - 0.35 \\ &= 0.75 \end{aligned}$$

Here is a visual interpretation of **A or B** and **A and B** events using Venn Diagram.

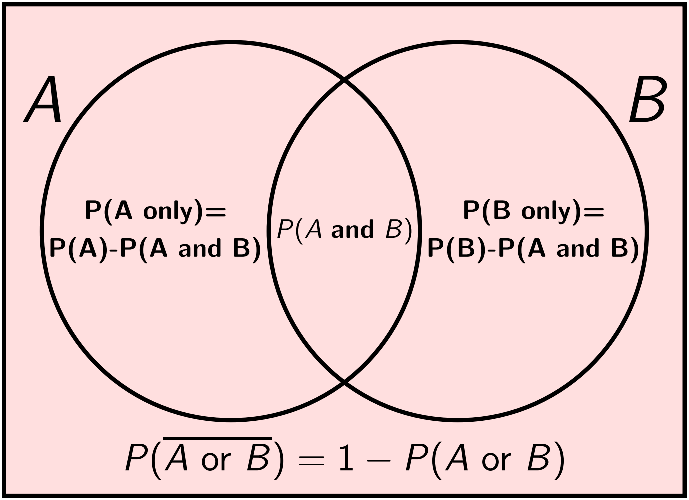




Let's look at A only, B only, and $\overline{A \text{ or } B}$ regions.



Now what about $P(A \text{ only})$, $P(B \text{ only})$, and $P(\overline{A \text{ or } B})$?



Example:

Given: $P(A) = 0.55, P(B) = 0.65,$ and $P(A \text{ and } B) = 0.25,$
Construct the Venn Diagram using the given information.

Solution:

We first find $P(A \text{ only})$, $P(B \text{ only})$, $P(A \text{ or } B)$, and $P(\overline{A \text{ or } B})$.

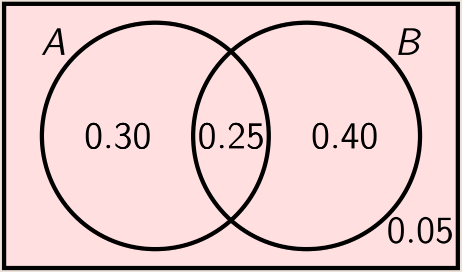
$$\begin{aligned} P(A \text{ only}) &= P(A) - P(A \text{ and } B) \\ &= 0.55 - 0.25 \\ &= 0.30 \end{aligned}$$

$$\begin{aligned} P(B \text{ only}) &= P(B) - P(A \text{ and } B) \\ &= 0.65 - 0.25 \\ &= 0.40 \end{aligned}$$

Solution Continued:

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= 0.55 + 0.65 - 0.25 \\ &= 0.95 \end{aligned}$$

$$\begin{aligned} P(\overline{A \text{ or } B}) &= 1 - P(A \text{ or } B) \\ &= 1 - 0.95 \\ &= 0.05 \end{aligned}$$



What are **Mutually Exclusive Events**?

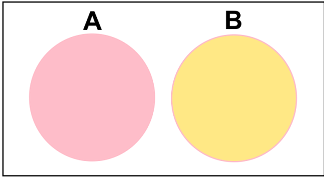
Two different events A and B, are called **Mutually Exclusive Events** or **Disjoint Events** if they cannot happen at the same time.

What is the property of two **Disjoint Events**?

For any two disjoint (mutually exclusive) events A and B,

$$P(A \text{ and } B) = 0$$

Here is a visual interpretation of two **Disjoint Events** using Venn Diagram.



Example:

Given: $P(A) = 0.55, P(B) = 0.25$, and A and B are mutually exclusive events, find $P(A \text{ or } B)$.

Solution:

Using the addition rule, we get

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= 0.55 + 0.25 - 0 \\ &= 0.8 \end{aligned}$$

Example:

A survey was conducted on 100 randomly selected adults to determine on what day of the week they did not report to work. The result of the survey is given below.

	Mon	Tues	Wed	Thurs	Fri
Females	15	5	3	7	15
Males	19	6	3	5	22

If one person is randomly selected from this group, find the probability that this person

- is a male.
- missed work on Friday.
- is a male and missed work on Friday.
- is a male or missed work on Friday.

Solution:

Let's begin by finding the total for each category.

	Mon	Tues	Wed	Thurs	Fri	Total
Females	15	5	3	7	15	45
Males	19	6	3	5	22	55
Total	34	11	6	12	37	100

- Male $\Rightarrow P(Male) = \frac{55}{100} = \frac{11}{20}$
- Friday $\Rightarrow P(Friday) = \frac{37}{100}$
- Male and Friday $\Rightarrow P(Male \text{ and } Friday) = \frac{22}{100} = \frac{11}{50}$
- Male or Friday $\Rightarrow P(Male \text{ or } Friday) = \frac{55}{100} + \frac{37}{100} - \frac{22}{100} = \frac{70}{100} = \frac{7}{10}$